**Program:** RFEM 5, RF-FORM-FINDING

**Category:** Large Deformation Analysis, Isotropic Linear Elasticity, Shell

**Verification Example:** 0206 – Balloon – Prestressed Membrane

# 0206 – Balloon – Prestressed Membrane

## Description

A spherical balloon membrane is filled with gas with atmospheric pressure $p_0$ and volume V according to **Figure 1** (These values are used for FE model definition only). Determine the over-pressure $p$ inside the balloon due to the given isotropic membrane prestress $n$. The add-on module RF-FORM-FINDING is used for this purpose. Elastic deformations are neglected both in RF-FORM-FINDING and in analytical solution, self-weight is also neglected in this example. The problem is described by the following set of parameters.

| | | | | | |
|---|---|---|---|---|---|
| Material | Polymer | Modulus of Elasticity | $E$ | 692.000 | MPa |
| | | Poisson's Ratio | $\nu$ | 0.442 | – |
| Geometry | | Balloon Radius | $r$ | 1.500 | m |
| | | Membrane Thickness | $t$ | 1.000 | mm |
| Load | | Atmospheric Pressure | $p_0$ | 0.100 | MPa |
| | | Gas Volume | $V$ | 14.137 | $m^3$ |
| | | Membrane Prestress | $n$ | 1.000 | kN/m |

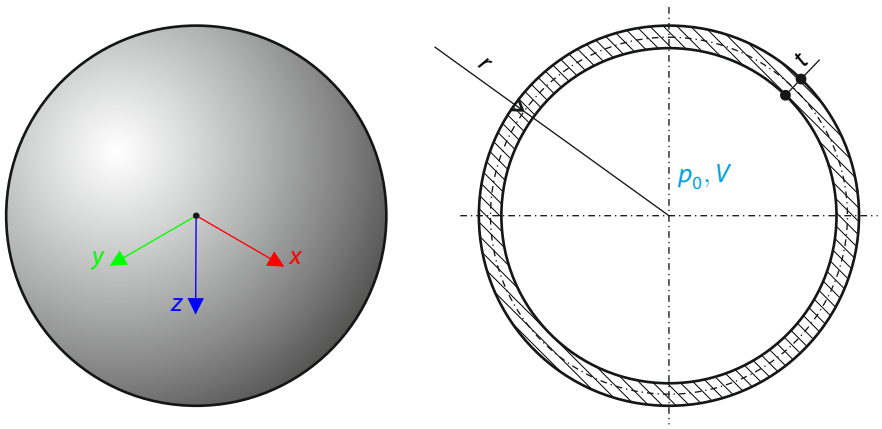


**Figure 1:** Problem Sketch

## Analytical Solution

The stress state of a membrane can generally be expressed by means of the Laplace equation

$$\frac{\sigma_1}{r_1} + \frac{\sigma_2}{r_2} = \frac{p}{t}, \qquad (206 - 1)$$

where $\sigma_1, \sigma_2$ are stresses in meridian and parallel direction respectively, $r_1, r_2$ are the radii in the corresponding directions and $p$ is the inner overpressure. This equation can be transformed into the form with normal forces

$$\frac{n_1}{r_1} + \frac{n_2}{r_2} = p. \qquad (206 - 2)$$

In case of a spherical membrane, where $n_1 = n_2 = n$ and $r_1 = r_2 = r$, **(206 – 2)** can be reduced and the desired overpressure calculated as

$$p = \frac{2n}{r} \approx 1333.333 \text{ Pa}. \qquad (206 - 3)$$

## RFEM 5 Settings

- Modeled in RFEM 5.16.01
- The element size is $l_{FE} = 0.200$ m
- Isotropic linear elastic material model is used

## Results

| Structure Files | Program | Modul |
|---|---|---|
| 0206.01 | RFEM 5 | RF-FORM-FINDING |

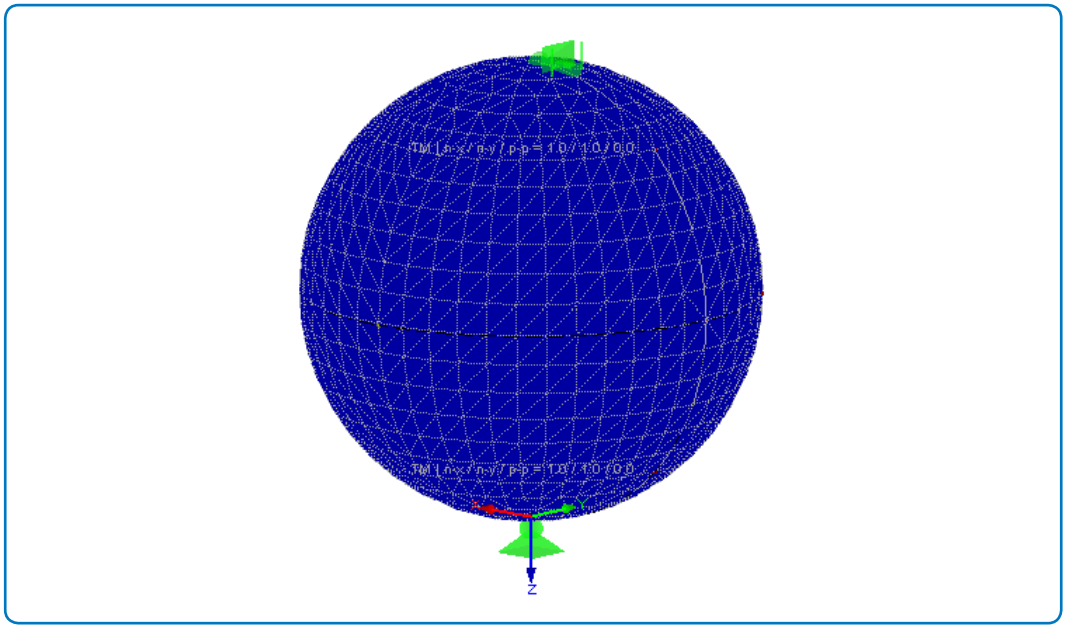


**Figure 2:** Model and results in RFEM 5

| Analytical Solution | RFEM 5 – RF-FORM-FINDING | |
|---|---|---|
| $p$ [Pa] | $p$ [Pa] | Ratio [-] |
| 1333.333 | 1334.918 | 1.001 |