Verification Example

Program: RFEM 5, RF-FORM-FINDING

Category: Large Deformation Analysis, Isotropic Linear Elasticity, Shell

Verification Example: 0207 – Catenoid

0207 – Catenoid

Description

A cylindrical membrane is stretched by means of isotropic prestress n according to **Figure 1**. Find the final minimal shape of the membrane – catenoid. Determine the maximum radial deflection of the membrane u. The add-on module RF-FORM-FINDING is used for this purpose. Elastic deformations are neglected both in RF-FORM-FINDING and in analytical solution, also self-weight is neglected in this example. The problem is described by the following set of parameters.

Material	Polymer	Modulus of Elasticity	E	692.000	MPa
		Poisson's Ratio	ν	0.442	–
Geometry		Height	h	1.000	m
		Diameter	d	2.000	m
		Thickness	t	1.000	mm
Load		Prestress	n	1.000	kN/m

Remark: The solution does not depend on the thickness t but it has to be set up in RFEM 5 model.

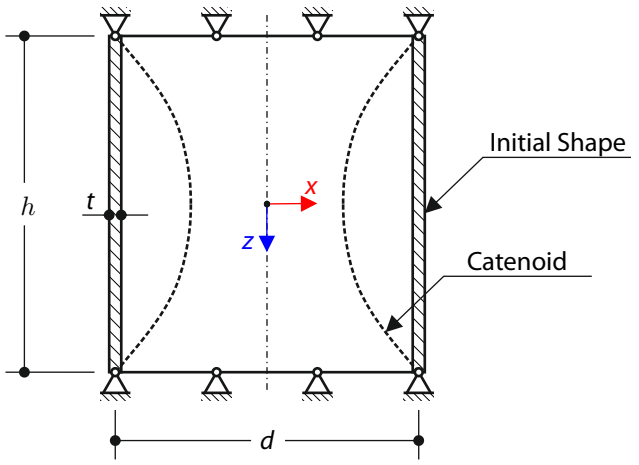

Figure 1: Problem sketch

Analytical Solution

A catenoid is a minimal surface arising by the revolution of a catenary curve about an axis. Catenary curve is described in Verification example 0079 in detail, [1]. Catenoid is described by means of following set of parametric equations

$$x = a \cosh\left(\frac{u}{a}\right) \cos(v), \tag{207 – 1}$$

$$y = a \cosh\left(\frac{u}{a}\right) \sin(v), \tag{207 – 2}$$

$$z = u, \tag{207 – 3}$$

where $v \in [-\pi, \pi)$ and a is a non-zero real constant. For the bottom circle it holds that $u = \frac{h}{2}$ and due to the axisymmetry, an arbitrary parameter v can be taken, in thic case $v = 0$. Then the equation **(207 – 1)** can be written as

$$\frac{d}{2} = a \cosh\left(\frac{h}{2a}\right). \tag{207 – 4}$$

The constant a has to be determined numerically. The constant a has the meaning of $x-$ or $y-$coordinate at $z = 0$. Thus, the maximum deflection u_x is

$$u = \frac{d}{2} - a \approx 151.662 \text{ mm}. \tag{207 – 5}$$

RFEM 5 Settings

- Modeled in RFEM 5.15.01
- The element size is $l_{FE} = 0.100$ m
- Isotropic linear elastic material model is used

Results

Structure Files	Program	Modul
0207.01	RFEM 5	RF-FORM-FINDING

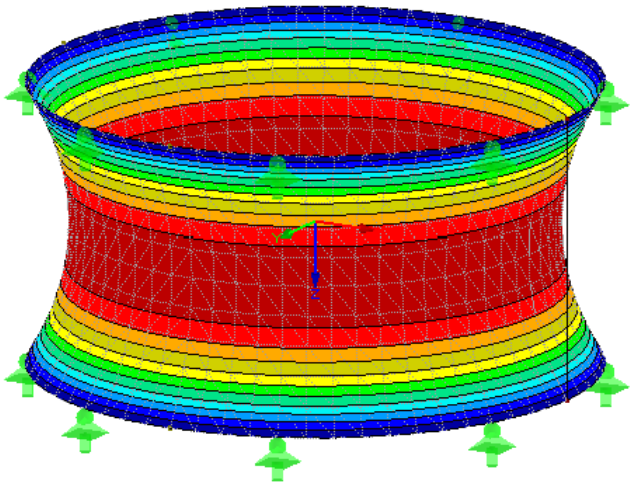

Figure 2: Result membrane shape (catenoid) in RFEM 5

Analytical Solution	RFEM 5 – RF-FORM-FINDING	
u [mm]	u [mm]	Ratio [-]
151.662	151.791	1.001

References

[1] DLUBAL SOFTWARE GMBH, *Verification Example 0079 – Catenary*. 2018.