**Program:** RFEM 5

**Category:** Geometrically Linear Analysis, Isotropic Nonlinear Elasticity, Plate, Solid

**Verification Example:** 0014 – Loaded Plastic Beams with Decaying Stress-Strain Curve

# 0014 – Loaded Plastic Beams with Decaying Stress-Strain Curve

## Description

Four columns with width *d*, depth *d*, height $h$ and distance *d* between them are oriented in the direction of the *z*-axis. They are fixed at the bottom and connected by the rigid block at the top. Block is loaded by the pressure *p* in the *z*-direction and modeled by an elastic material with high modulus of elasticity $E_r$. Outer columns are modeled by linear elastic material and inner columns by a stress-strain diagram with the decaying dependence for $\varepsilon > \varepsilon_0$, where the limit is set to $\varepsilon_0 = 0.005$. The stress-strain curves for each material and the overall stress-strain curve are depicted on **Figure 1**. Assuming only small deformations theory and neglecting structure's self-weight, determine its maximum deflection.

| | | | | | |
|---|---|---|---|---|---|
| Material | Columns | Modulus of Elasticity | $E_1$ | 50.000 | GPa |
| | | Poisson's Ratio | $\nu$ | 0.000 | – |
| | Inner Columns - Stress-Strain Curve | Modulus of Elasticity of the 2nd branch | $E_2$ | 40.000 | GPa |
| | | Strain Limit | $\varepsilon_0$ | 0.005 | – |
| | Block | Modulus of Elasticity | $E_r$ | 20000.000 | GPa |
| | | Poisson's Ratio | $\nu$ | 0.000 | – |
| Geometry | Column | Height | $h$ | 1.000 | m |
| | | Depth<br>Width<br>Distance | $d$ | 0.100 | m |
| | Block | Height<br>Depth | $d$ | 0.100 | m |
| | | Width | $7d$ | 0.700 | m |
| Load | | Pressure | $p$ | 0.158 | GPa |

## Analytical solution

### Linear Analysis

Applying Hook's law and assuming that the acting loading will be divided equally to the each column, simple formula for the maximum deflection can be derived:

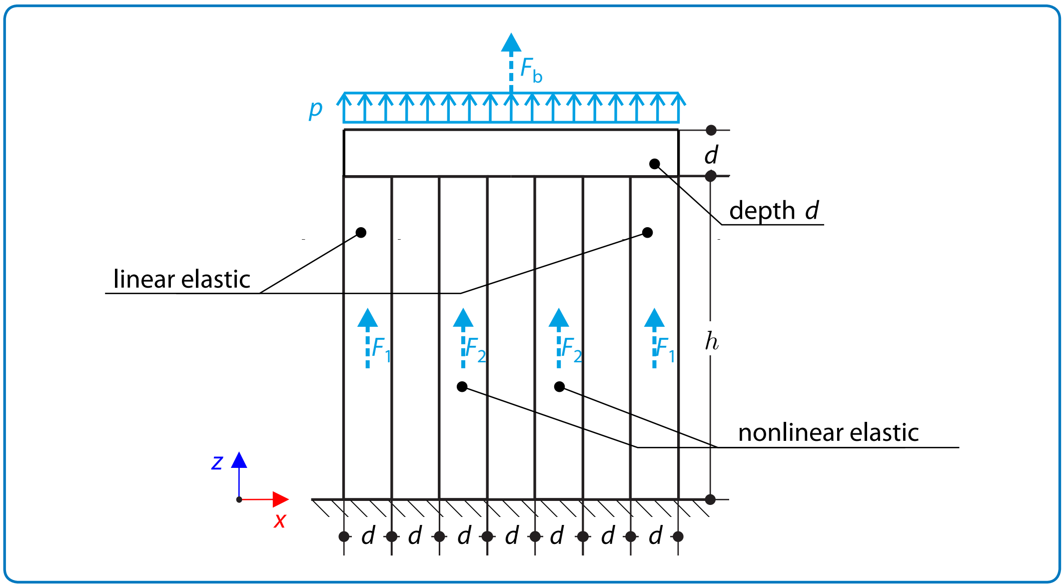


**Figure 1:** Problem sketch

$$u_{z,\max} = \varepsilon h = \frac{\sigma_{\text{column}}}{E_1} h = \frac{A_{\text{block}}}{A_{\text{column}}} \frac{ph}{4E_1} = \frac{7ph}{4E_1} = 5.530 \text{ mm} \tag{14 – 1}$$

where $A_{\text{block}}$ and $A_{\text{column}}$ is cross section of block and one column respectively.

## Nonlinear Analysis

The maximum deflection of the structure can be obtained by the following formula:

$$u_{z,\max} = \varepsilon h \tag{14 – 2}$$

where $\varepsilon$ is the total strain of structure, which can be derived from the condition of force equilibrium:

$$2(F_1 + F_2) = F_b \tag{14 – 3}$$

where $F_b$ is the force acting on the block due to the applied pressure:

$$F_b = 7d^2 p \tag{14 – 4}$$

and $F_1$, $F_2$ are forces acting on the outer and inner column respectively:

$$F_1 = E_1 \varepsilon d^2 \tag{14 – 5}$$

$$F_2 = [(E_1 + E_2)\varepsilon_0 - E_2 \varepsilon]\, d^2 \tag{14 – 6}$$

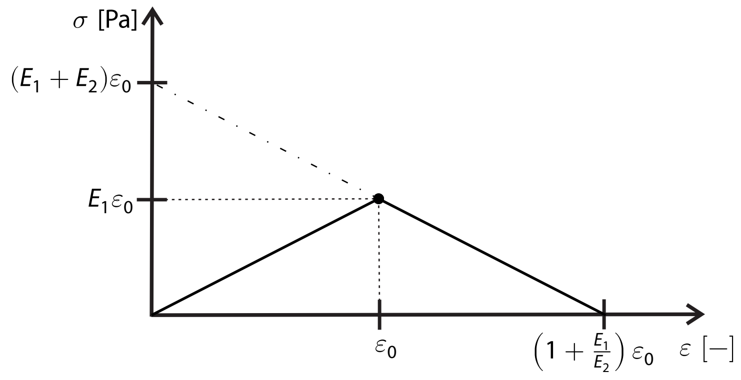


**Figure 2:** Material stress-strain curve

Substituting the equations **(14 – 4)** - **(14 – 6)** into the equation **(14 – 3)**, formula for the total strain of the structure can be derived:

$$\varepsilon = \left[\frac{7p}{2} - (E_1 + E_2)\varepsilon_0\right]\frac{1}{E_1 - E_2} \tag{14 – 7}$$

The maximum deflection of the structure can be finally obtained by substituting into the equation **(14 – 2)**:

$$u_{z,\text{max}} = \left[\frac{7p}{2} - (E_1 + E_2)\varepsilon_0\right]\frac{h}{E_1 - E_2} = 10.300 \text{ mm} \tag{14 – 8}$$

## RFEM 5 Settings

- Modeled in version RFEM 5.05.0030
- The element size is $l_{\text{FE}} = 0.050$ m
- Geometrically linear analysis is considered
- The number of increments is 1
- The Mindlin plate theory is used
- Nonsymmetric direct solver is used

## Results

| Structure File | Entity | Material Model |
|---|---|---|
| 0014.01 | Solid | Isotropic Nonlinear Elastic 2D/3D |
| 0014.02 | Plate | Isotropic Nonlinear Elastic 2D/3D |
| 0014.03 | Solid | Isotropic Linear Elastic |
| 0014.04 | Plate | Isotropic Linear Elastic |

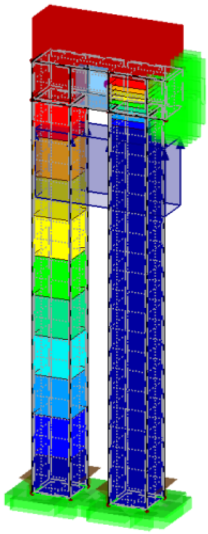

**Figure 3:** Model in RFEM 5

## Linear Analysis

| Analytical Solution | RFEM 5 (Solid - Isotropic Linear Elastic) | | RFEM 5 (Plate - Isotropic Linear Elastic) | |
|---|---|---|---|---|
| $u_{z,\max}$ [mm] | $u_{z,\max}$ [mm] | Ratio [-] | $u_{z,\max}$ [mm] | Ratio [-] |
| 5.530 | 5.530 | 1.000 | 5.530 | 1.000 |

## Nonlinear Analysis

| Analytical Solution | RFEM 5 (Solid - Isotropic Nonlinear Elastic 2D/3D) | | RFEM 5 (Plate - Isotropic Nonlinear Elastic 2D/3D) | |
|---|---|---|---|---|
| $u_{z,\max}$ [mm] | $u_{z,\max}$ [mm] | Ratio [-] | $u_{z,\max}$ [mm] | Ratio [-] |
| 10.300 | 10.300 | 1.000 | 10.300 | 1.000 |