Verification Example

Program: RFEM 5, RF-GLASS, RF-IMP

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Glass, Plate, Solid

Verification Example: 0032 – Glass-Foil-Glass-Gas-Glass Plate

0032 – Glass-Foil-Glass-Gas-Glass Plate

Description

A composite plate, consisting of three glass layers, one foil layer and the inner space with the dry air is fully fixed and loaded with the variable temperature. Neglecting it's self-weight, determine plate's maximum deflection u_z.

Material	Glass	Modulus of Elasticity	$E_1 = E_3$	70000.000	MPa
		Poisson's Ratio	$\nu_1 = \nu_3$	0.230	–
	Foil	Modulus of Elasticity	E_2	3.000	MPa
		Poisson's Ratio	ν_2	0.499	–
Geometry	Plate	Height	a	1000.000	mm
		Internal Radius of Curvature	r	3000.000	mm
		Central Angle	α	30.000	°
	Glass pane	Thickness	t_1	5.000	mm
	Inner space	Thickness	t_2	12.00	mm
	Glass pane	Thickness	t_3	5.000	mm
	Foil	Thickness	t_4	0.760	mm
	Glass pane	Thickness	t_5	5.000	mm
Load	Factory	Temperature	T_p	0	°C
		Atmospheric Pressure	$p_{p,met}$	0.101	MPa
	Mount	Temperature	T_1	25.000	°C
		Atmospheric Pressure	$p_{out,met}$	0.101	MPa
		Height Difference from Factory	$\triangle H$	0	

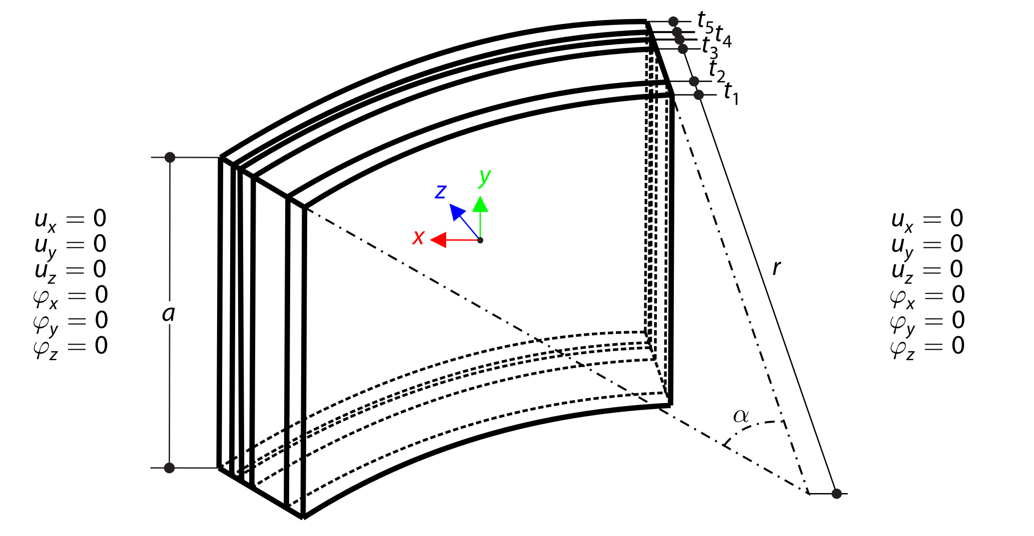

Figure 1: Problem sketch

Analytical Solution

Because of the excessive time demands of the analytical solution, RF-IMP Module will be used to assist with the calculations. In RFEM 5, all three glass layers and one foil layer were modeled as solids and gas layer was replaced by the surface load p, which could be calculated from the thermal state equation for ideal gases:

$$\frac{p_\mathrm{p} V_{01}}{T_\mathrm{p}} = \frac{p_1 V_1}{T_1} = \frac{p_1 \left(V_{01} + C_\mathrm{V}(p_1 - p_\mathrm{out})\right)}{T_1} \tag{32 – 1}$$

where p_p and p_out is the initial gas pressure and the external gas pressure respectively and with no difference between manufacturing and mount atmospheric pressure and no change in the sea level, they are both equal to the atmospheric pressure, V_{01} is the initial volume

$$V_{01} = a \frac{\alpha \pi r}{180} t_2 \tag{32 – 2}$$

and C_V is the ductility of glass plates

$$C_\mathrm{V} = \frac{V(p)}{p} \tag{32 – 3}$$

where $V(p)$ is the volume change of a given glass layer due to the pressure p.

Equation **(32 – 1)** can be expressed in the quadratic form:

$$C_\mathrm{V} p_1^2 + (V_{01} - C_\mathrm{V} p_\mathrm{out}) p_1 - \frac{p_\mathrm{p} V_{01} T_1}{T_\mathrm{p}} = 0 \tag{32 – 4}$$

Solving this quadratic equation, the expression for the internal gas pressure at mount p_1 can be obtained:

$$p_1 = \frac{C_V p_{out} - V_{01} + \sqrt{(V_{01} - C_V p_{out})^2 + 4C_V \frac{p_p V_{01} T_1}{T_p}}}{2C_V} \tag{32 – 5}$$

The factor C_V depends on the support type, the dimensions and the stiffness of the glass panes and can be expressed by the following formula:

$$C_V = C_{V1} + C_{V2} \tag{32 – 6}$$

where C_{V1} is the ductility of layer 1

$$C_{V1} = \frac{V_1}{p} = \frac{1}{p} \int_0^a \int_0^b u_{z,\text{layer1}}(x, y) \text{d}x\text{d}y \tag{32 – 7}$$

and C_{V2} is the ductility of layer 3

$$C_{V2} = \frac{V_2}{p} = \frac{1}{p} \int_0^a \int_0^b u_{z,\text{layer3}}(x, y) \text{d}x\text{d}y \tag{32 – 8}$$

where V_n and $u_{z,n}$ are layer's volume change and deformation respectively. Because the factor C_V depends on the pressure $p = p_1 - p_{out}$, the calculation is iterative. Following table contains data from each iteration step, where layer's volume changes V_1 and V_2 were determine from the deformations of individual points in the model obtained by RF-IMP Add-on Module.

Iteration step	p_1	p	$u_{z,\text{layer1}}$	$u_{z,\text{layer3}}$	V_1	V_2	C_V
	[Pa]	[Pa]	[mm]	[mm]	[mm^3]	[mm^3]	[mm^3/Pa]
1st	106000	5000	-0.178	0.084	1.50×10^5	0.75×10^5	44.95
2nd	108351	7351	-0.265	0.123	2.21×10^5	1.10×10^5	45.03
3rd	108349	7349	-0.265	0.123	–	–	–

As can be seen from the previous table, maximal deformations of layers 1 and 3 ($u_{z,1}$ and $u_{z,2}$ respectively) are identical for the 2nd and 3rd iteration step and can be understood as final values.

RFEM 5 Settings

- Modeled in version RFEM 5.03.0050
- The element size is $l_{FE} = 0.050$ m
- Geometrically linear analysis is considered
- The number of increments is 1
- The Mindlin plate theory is used
- Isotropic linear elastic material model is used
- Coupling of layers is considered
- Plate's local z-axis is oriented out of the circular arc.

Results

Structure File	Program	Description
0032.01	RF-GLASS	Problem Solution
0032.02	RFEM 5	Last Iteration step of the Theoretical Solution

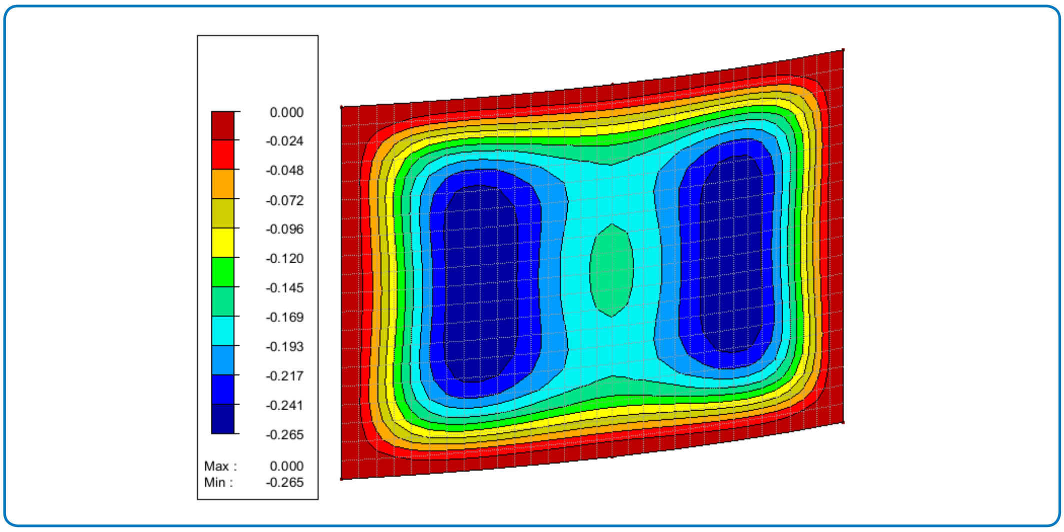

Figure 2: RF-GLASS Solution for layer 1

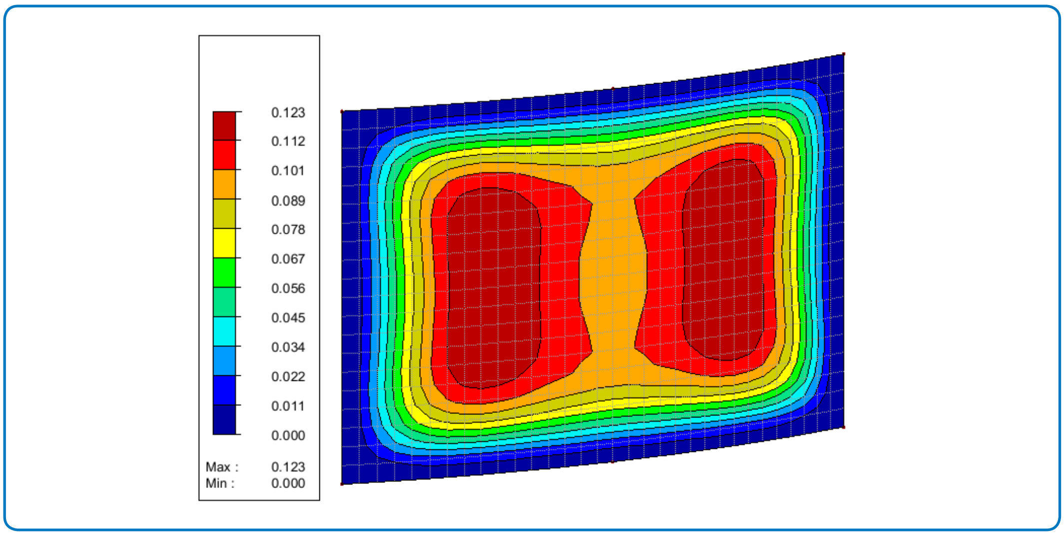

Figure 3: RF-GLASS Solution for layer 3

As can be seen from the following comparison, an excellent agreement of RF-GLASS output with the analytical solution was achieved.

Quantity	Analytical solution	RF-GLASS	
	[mm]	[mm]	Ratio [-]
$u_{z,\text{layer1}}$	−0.265	−0.265	1.000
$u_{z,\text{layer3}}$	0.123	0.123	1.000