## Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Structural Nonlin-

 earity, Member
## Verification Example: 0055 - Scaffolding Nodal Support

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## Description

Consider a rigid scaffolding tube, fixed at the bottom using the Scaffolding Nodal Support and loaded by both a moment $\boldsymbol{M}$ and a force of magnitude $P_{z}$. Self-weight is not considered. The problem is described by the following set of input parameters.

| Material | Steel | Modulus of Elasticity | E | 210.000 | GPa |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Geometry | Scaffolding tube | Length | L | 1.000 | m |
|  |  | Cross-Section |  | RO244.5×25 |  |
| Loading | Scaffolding tube | Moment | $M_{1{ }_{\text {x }}}$ | 0.000 | kNm |
|  |  |  | $M_{11_{r}}$ | 1.000 | kNm |
|  |  |  | $M_{2 \times}$ | $\frac{\sqrt{2}}{2}$ | kNm |
|  |  |  | $M_{2 r}$ | $\frac{\sqrt{2}}{2}$ | kNm |
|  |  | Force | $P_{Z}$ | 50.000 | kN |
| Support properties | Scaffolding <br> Nodal <br> Support | Maximal Eccentricity | $e_{\text {max }}$ | 0.025 | m |
|  |  | Stiffness | C | 20.000 | kNm/rad |

Example presumptions:

- Boundary conditions $u_{x}=u_{y}=u_{z}=\varphi_{z}=0$ for $z=0$
- The behaviour of the Scaffolding Nodal Support depends on a M-Phi diagram, where $M=e P_{z}$ in accordance with ČSN EN 12811-1 norm [1]. See Figure 2.

Consider infinitely rigid beam and determine maximum radial deflection $u_{r, \max }=\sqrt{u_{x, \max }{ }^{2}+u_{y, \max }{ }^{2}}$ of the structure in two cases:

- Firstly, consider a moment $\boldsymbol{M}_{1}=\left[M_{1_{x}}, M_{1_{y}}, 0\right]=[0,1,0]$ acting around Y -axis.
- Secondly, consider more general moment $\boldsymbol{M}_{2}=\left[M_{2_{x}}, M_{2_{2}}, 0\right]=\left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right]$. See Figure 1 for orientation.

Determine the above quantities using a beam with RO244.5×25 cross-section. Such beam is a suitable model of a physically unrealistic ideally rigid beam.

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Figure 1: Problem Sketch and Solution


Figure 2: Relationship between magnitude of moment $M=e P_{z}$ and angle $\varphi$ for $P_{z}=1 \mathrm{~N}$. In this case the value of moment equals the value of Eccentricity. The same diagram is used in RFEM to define a non-linearity of Scaffolding Nodal Support.

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Figure 3: Relationship between magnitude of moment $M=e P_{z}$ and angle $\varphi$ for $P_{z}=50 \mathrm{kN}$.

## Analytical Solution

The problem presents a simplified model where the tube is considered rigid. Thus we can get the solution of both cases immediately by reading the values from Figure 3.

$$
\begin{align*}
& \varphi_{1}=\frac{\left|\boldsymbol{M}_{1}\right|}{C}=\frac{1}{20}=0.05 \mathrm{rad}  \tag{55-1}\\
& \varphi_{2}=\frac{\left|\boldsymbol{M}_{2}\right|}{C}=\frac{1}{20}=0.05 \mathrm{rad} \tag{55-2}
\end{align*}
$$

This in turn yields the maximal radial deflection $u_{r, \max }$.

$$
\begin{align*}
& u_{r, \max }\left(\boldsymbol{M}_{1}\right) \approx 1 \varphi_{1} \approx 50.00 \mathrm{~mm} \\
& u_{r, \max }\left(\boldsymbol{M}_{2}\right) \approx 1 \varphi_{2} \approx 50.00 \mathrm{~mm}
\end{align*}
$$

Please note that we have used 1st order theory approximation.

## RFEM 5 Settings

- Modeled in RFEM 5.04.0024
- The element size is $I_{\mathrm{FE}}=0.50 \mathrm{~m}$
- Geometrically linear analysis is considered


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- The number of increments 5
- Support Conditions: Diagram - FZ ${ }^{\prime} /$ PhiX $^{\prime}$ PhiY ${ }^{\prime}$ is given by Figure 2


## Results

| Structure File | Material Model | Description |
| :---: | :---: | :---: |
| 0055.01 | Rigid | Default Z-axis |
| 0055.02 | Rigid | Inverted Z-axis |
| 0055.03 | Isotropic Linear Elastic | RO244.5x25 cross-section |

## Modeled using rigid beams

| Load Case | Analytical solution* | RFEM 5 |  |
| :---: | :---: | :---: | :---: |
|  | $u_{r, \max }$ <br> $[\mathrm{~mm}]$ | $u_{r, \max }$ <br> $[\mathrm{~mm}]$ | Ratio <br> $[-]$ |
| Moment $\boldsymbol{M}_{1}$ | 50.00 | 50.00 | 1.000 |
| Moment $\boldsymbol{M}_{2}$ | 50.00 | 50.00 | 1.000 |

Modeled using rigid beams and inverted Z-axis

| Load Case | Analytical solution* | RFEM 5 |  |
| :---: | :---: | :---: | :---: |
|  | $u_{r, \max }$ <br> $[\mathrm{~mm}]$ | $u_{r, \max }$ <br> $[\mathrm{~mm}]$ | Ratio <br> $[-]$ |
| Moment $\boldsymbol{M}_{1}$ | 50.00 | 50.00 | 1.000 |
| Moment $\boldsymbol{M}_{2}$ | 50.00 | 50.00 | 1.000 |

Modeled using beams with RO244.5x25 cross-section

| Load Case | Analytical solution* | RFEM 5 |  |
| :---: | :---: | :---: | :---: |
|  | $u_{r, \max }$ <br> $[\mathrm{~mm}]$ | $u_{r, \max }$ <br> $[\mathrm{~mm}]$ | Ratio <br> $[-]$ |
| Moment $\boldsymbol{M}_{1}$ | 50.00 | 50.02 | 1.000 |
| Moment $\boldsymbol{M}_{2}$ | 50.00 | 50.02 | 1.000 |

* Analytical solution was derived using rigid beams and 1st order theory in all cases


## References

[1] ČSN EN 12811-1, Dočasné stavební konstrukce - Část 1: Pracovní lešení - Požadavky na provedení a obecný návrh. 2004.

