Program: RFEM 5

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Plate

Verification Example: 0071 – Clamped Elliptic Plate Under Transversal Load

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Description

An elliptic plate with clamped boundary is subjected to a uniformly distributed transversal load p. Assuming small deformation theory and neglecting self-weight, determine the maximum out-of-plane deflection u_{max} of the plate.

Material	Linear Elastic	Modulus of Elasticity	E	50.000	GPa
		Poisson's Ratio	ν	0.200	–
Geometry	Ellipse	Thickness	t	0.200	m
		Semi-Major Axis Length	a	2.000	m
		Semi-Minor Axis Length	b	1.000	m
Load		Pressure	p	10.000	MPa

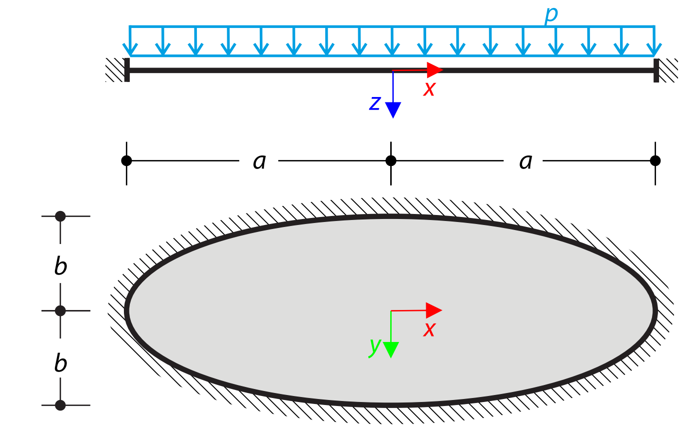

Figure 1: Problem sketch

Analytical Solution

The governing differential equation of a plate subjected to a distributed transversal load is related to the biharmonic operator $\nabla^2\nabla^2 u$, more precisely

$$\nabla^2\nabla^2 u = \frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4} = \frac{p}{D} \qquad (71-1)$$

where u is the out-of-plane deformation of the plate and D the flexural rigidity of the plate

$$D = \frac{Et^3}{12(1-\nu^2)} \qquad (71-2)$$

The biharmonic equation (71 – 1) is augmented with the clamped boundary condition

$$u = 0 \tag{71 – 3}$$

$$\frac{\partial u}{\partial x} = 0 \tag{71 – 4}$$

$$\frac{\partial u}{\partial y} = 0 \tag{71 – 5}$$

According to [1], the deflection surface of the elliptic plate is described by

$$u(x, y) = C\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)^2 \tag{71 – 6}$$

where a and b are the semi-major and semi-minor axes of the ellipse, respectively. The constant C can be obtained from substituting (71 – 6) into (71 – 1), which yields

$$\frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4} = 8C\left[3\left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2 - \frac{4}{a^2 b^2}\right] = \frac{p}{D} \tag{71 – 7}$$

hence

$$C = \frac{p}{8D\left[3\left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2 - \frac{4}{a^2 b^2}\right]} \tag{71 – 8}$$

It can be seen from the general solution (71 – 6) that the plate will be deflected the most at its centroid where $x = 0$ and $y = 0$, that is

$$u_{\text{max}} = u(0, 0) = C = \frac{3p(1 - \nu^2)}{2Et^3\left[3\left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2 - \frac{4}{a^2 b^2}\right]} \approx 9.763 \text{ mm} \tag{71 – 9}$$

RFEM 5 Settings

- Modeled in version RFEM 5.06.3039
- The element size is $l_{\text{FE}} = 0.01$ m
- Geometrically linear analysis is considered
- Number of increments is 1
- Kirchhoff plate theory is used

Results

Structure File	Program
0071.01	RFEM 5

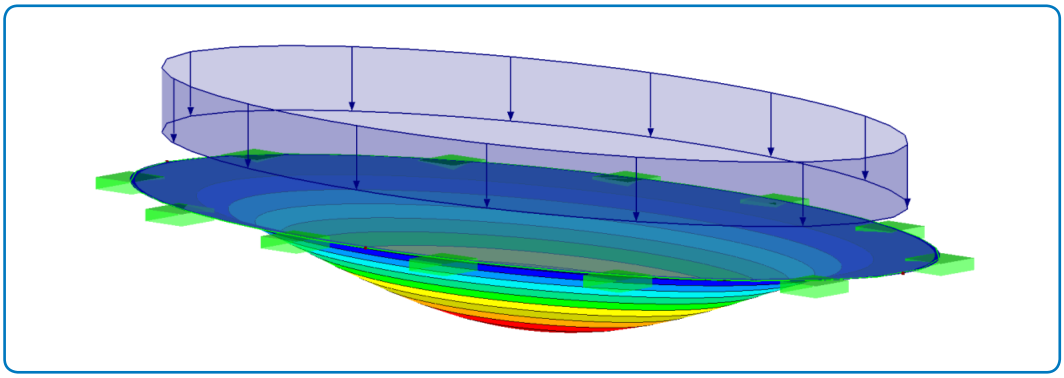

Figure 2: RFEM 5 Solution

As can be seen from the table below, excellent agreement of numerical output with the analytical result was achieved.

Analytical Solution	RFEM 5	
u_{max} [mm]	u_{max} [mm]	Ratio [-]
9.763	9.763	1.000

References

[1] SZILARD, R. *Theories and Application of Plate Analysis: Classical Numerical and Engineering Method*. Hoboken, New Jersey.