Program:	**RFEM 5, RF-DYNAM Pro**
Category:	**Isotropic Plasticity, Dynamics, Member**
Verification Example:	**0124 – Plastic Material Oscillations**

0124 – Plastic Material Oscillations

Description

This verification example is based on Verification Example 0122, see [1]. A single-mass system without damping is subjected to an axial loading force F. An ideal elastic-plastic material with characteristics is assumed, according to **Figure 1**. Determine the time course of the end-point deflection, velocity and acceleration. The problem is described by the following parameters.

Material	Plastic	Modulus of Elasticity	E	50.000	MPa
		Poisson's Ratio	ν	0.000	–
		Yield Strength	f_y	1.000	MPa
Geometry	Beam	Length	L	0.300	m
	Cross-section	Height	h	20.000	mm
		Width	b	20.000	mm
Load		Force	F	300.000	N
Mass			m	100.000	kg

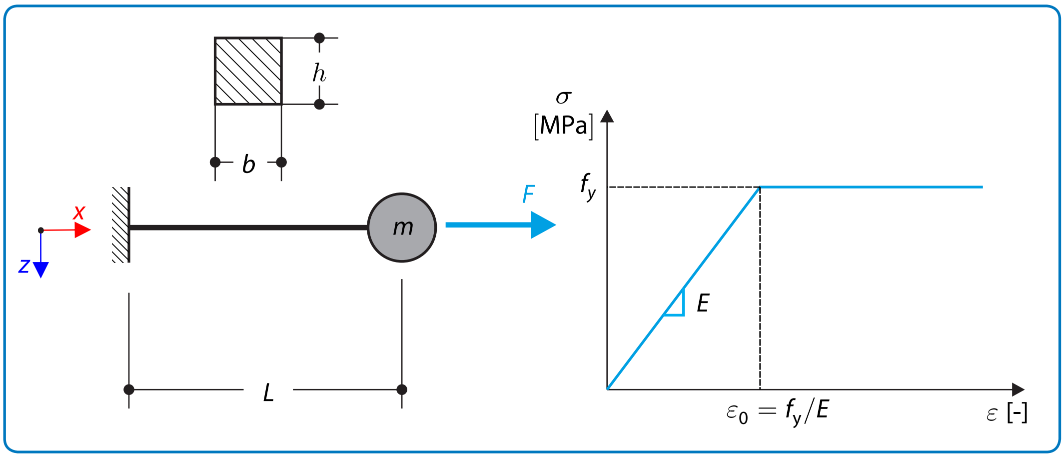

Figure 1: Problem sketch

Analytical Solution

Single-mass-system oscillations are described by the second-order differential equation.[1] In case of nonlinear elastic-plastic behaviour, this equation has to be divided due to the change of stiffness (only tension of the beam is considered)

[1] $\dot{u}_x$ and $\ddot{u}_x$ denote the first and second time derivative of u_x, respectively.

$$m\ddot{u}_x + ku_x = F, \qquad u_x \leq \frac{f_y L}{E}, \tag{124 – 1}$$

$$m\ddot{u}_x = F - f_y A, \qquad u_x \in \left(\frac{f_y L}{E}, \frac{2 f_y L}{E}\right), \tag{124 – 2}$$

$$m\ddot{u}_x + k(u_x - 2\frac{f_y L}{E}) = F - f_y A, \qquad \text{for steady oscillations,} \tag{124 – 3}$$

where the stiffness k is defined by the modulus of elasticity E, cross-section area $A = bh$ and the length of the beam L

$$k = \frac{EA}{L}. \tag{124 – 4}$$

The set of equations **(124 – 1)**, **(124 – 2)** and **(124 – 3)** is further solved by means of Runge–Kutta method. For time behaviour of the deflection, velocity and acceleration see **Figure 2**. The specific values at test time 0.3 s are listed below

$$u_x(0.3) = 9.026 \text{ [mm]} \tag{124 – 5}$$

$$\dot{u}_x(0.3) = -5.063 \text{ [mm/s]} \tag{124 – 6}$$

$$\ddot{u}_x(0.3) = 0.984 \text{ [m/s}^2\text{]} \tag{124 – 7}$$

RFEM 5 Settings

- Modeled in RFEM 5.14.01
- The global element size is $l_{FE} = 0.2$ m

Results

Structure Files	Method
0124.01	Explicit Analysis
0124.02	Nonlinear Newmark Analysis

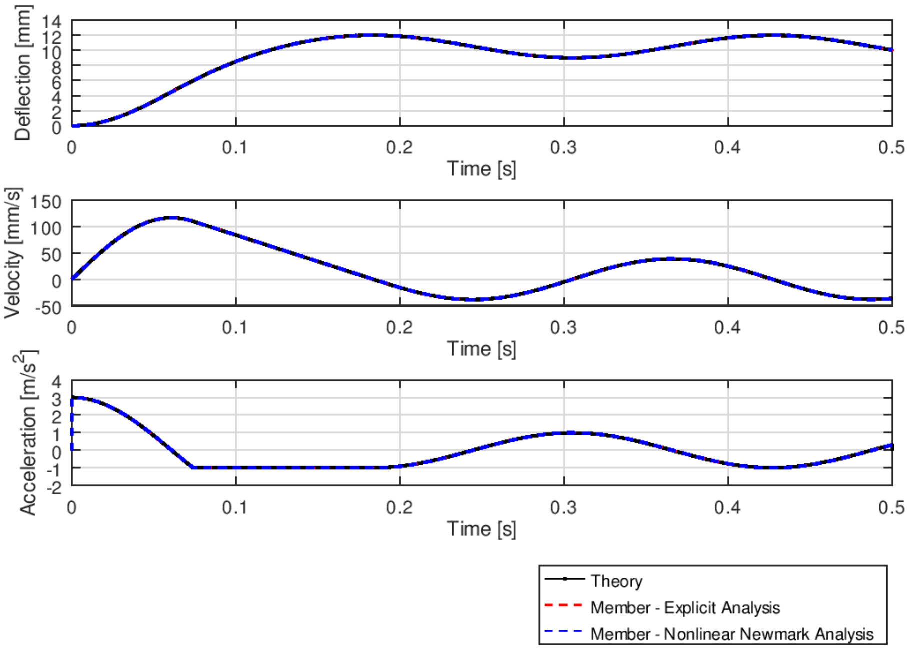

Figure 2: Results comparison

Model	Analytical Solution	RFEM 5	
	u_x [mm]	u_x [mm]	Ratio [-]
Explicit Analysis	9.026	9.029	1.000
Nonlinear Implicit Newmark Analysis		9.028	1.000

Model	Analytical Solution	RFEM 5	
	$\dot{u}_x$ [mm/s]	$\dot{u}_x$ [mm/s]	Ratio [-]
Explicit Analysis	-5.063	−5.353	1.057
Nonlinear Implicit Newmark Analysis		−5.397	1.066

Model	Analytical Solution	RFEM 5	
	$\ddot{u}_x$ [m/s^2]	$\ddot{u}_x$ [m/s^2]	Ratio [-]
Explicit Analysis	0.984	0.989	1.005
Nonlinear Implicit Newmark Analysis		0.989	1.005

References

[1] DLUBAL SOFTWARE GMBH, *Verification Example 0122 – Nonlinear Elastic Material Oscillations – Yielding*. 2018b.