1 Nodal Constraints in RFEM

1 Introduction

Nodal Constraints are explicit mathematical relations between several degrees of freedom associated with the user-defined set of nodes. The typical use of Nodal Constraints is to impose certain types of rigid-like behaviour, symmetric conditions or more general kinematic constraints.

2 Types of Constraint

In the present version of RFEM, two types of nodal constraint are implemented.

- Diaphragm Constraint
- Equal Condition Constraint

A Diaphragm Constraint enforces rigid-membrane behaviour and leaves out-of-plane deformations unaffected. This constraint presents a computationally efficient option for modeling floors, slabs, or structures that have very large in-plane stiffness.

An Equal Condition Constraint enforces that the same value for each selected degree of freedom is shared among all constrained nodes. Each Equal Condition Constraint can be used, among other things, to connect together different parts of the structure.
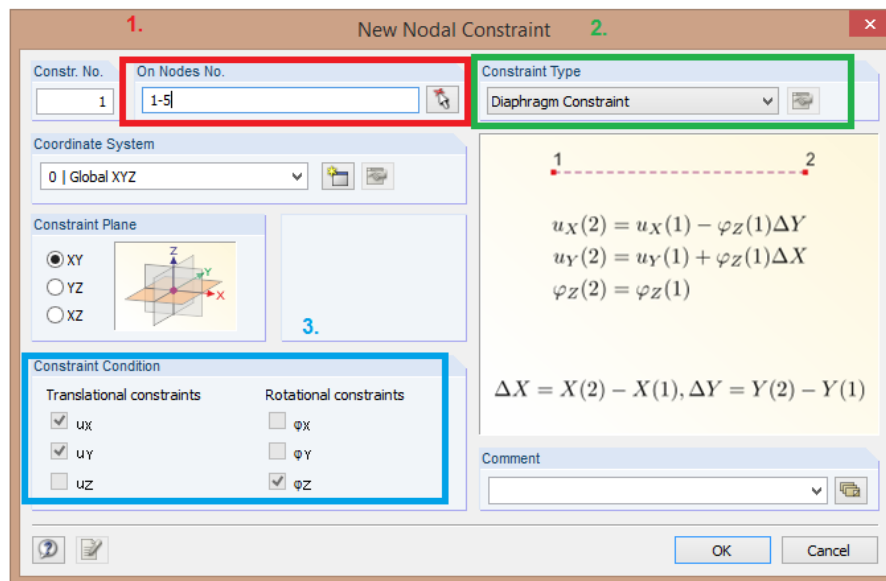
3 User Interface

Figure 1.1: 1.Node set definition, 2.Nodal Constraint type definition, 3. Active Degrees of freedom

In RFEM graphical user interface, Nodal Constraints are considered a new type. A new Nodal Support can be defined using a dialog window accessible from Data Tree in Project Navigator or Menu *Insert -> Model Data -> Nodal Constraints*. Moreover, a new Nodal Constrain can be defined in Model Data Table 1. A Nodal Constrain is defined by a set of constrained Nodes, a constraint type, a Coordinate System and a set of Degrees of freedom with respect to a particular Coordinate System. See Figure 1.1.

4 Mathematical implementation

Let us consider N constrained user-defined nodes.

- Diaphragm Constraint

  The following set of equalities, written with respect to global coordinate system XYZ, has to be satisfied for every two constrained points.

  $$u_X(j) = u_X(i) - \varphi_Z(i)\Delta Y$$
  $$u_Y(j) = u_Y(i) + \varphi_Z(i)\Delta X$$
  $$\varphi_Z(j) = \varphi_Z(i)$$
  $$\Delta X = X(j) - X(i)$$
  $$\Delta Y = Y(j) - Y(i)$$
  $$\forall i \neq j$$
  $$i,j = 1...\text{N}$$

- Equal Condition Constraint

  The following set of equalities , written with respect to global coordinate system XYZ, has to be satisfied for every two constrained points.

  $$u_X(j) = u_X(i)$$
  $$u_Y(j) = u_Y(i)$$
  $$u_Z(j) = u_Z(i)$$
  $$\varphi_X(j) = \varphi_X(i)$$
  $$\varphi_Y(j) = \varphi_Y(i)$$
  $$\varphi_Z(j) = \varphi_Z(i)$$
  $$\forall i \neq j$$
  $$i,j = 1...\text{N}$$

As explained, Nodal Constraints are explicit mathematical relations that due to their nature have to be enforced at the level of global stiffness matrix. There are two methods to achieve this goal.

- Penalty Method

- Lagrange Multipliers

The former technique uses penalty stiffness. Roughly speaking, each constraint is represented as fictitious elastic structural element that enforces the constraint approximately. For the latter method, additional unknowns (one for each constraint) are added to the original system of equations.
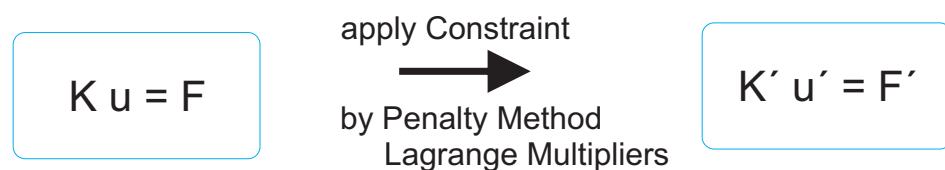
$$K\,u = F \quad \xrightarrow[\text{Lagrange Multipliers}]{\substack{\text{apply Constraint}\\ \text{by Penalty Method}}} \quad K'\,u' = F'$$

Figure 1.2: Constraint application.

Right now, only the Penalty method is fully implemented.

5 Remarks

The constraint is applied to the user-defined set of Nodes only. Nodes of finite element discretization that are not user-defined Nodes are not directly affected by the constraint.

Please note that neither a Diaphragm Constraint nor a Equal Condition Constrain is designed to capture true rigid-body behaviour. A Diaphragm Constraint enforces rigid-membrane behaviour but leaves out-of-plane deformations unaffected. Similarly, even if an Equal Condition constrain is applied on all 6 degrees of freedom (in 3D), it does not behave like rigid-body. The difference is that there is no coupling between rotations and the translations in terms of rigid-body mechanics.

See Verification Example 106 for an illustration of Diaphragm Constraint.