Program:	**RFEM 5, RF-LOAD-HISTORY**
Category:	**Geometrically Linear Analysis, Isotropic Nonlinear Elasticity, Isotropic Plasticity, Orthotropic Plasticity, Temperature Dependency, Member, Plate, Solid**
Verification Example:	**0008 – One-Dimensional Plasticity**

0008 – One-Dimensional Plasticity

Description

A three-dimensional block, made of elastic-plastic material, of height h, width w and depth d, is fixed on the both ends in z-direction and partly fixed in x and y-direction as it is shown in the **Figure 1**, all the rotations are constrained. The block's middle plane is subjected to the pressure load p. The block is made either from the orthotropic elastic-plastic material or from the isotropic elastic-plastic material, whose material properties are set do that both cases yield the equivalent behaviour. In the isotropic case, the problem is described by the following set of parameters.

Material	Isotorpic Elastic (upper block)	Modulus of Elasticity	E	11000.000	MPa
		Poisson's Ratio	ν	0.000	–
	Isotorpic Elastic-Plastic (lower block)	Modulus of Elasticity	E	11000.000	MPa
		Poisson's Ratio	ν	0.000	–
		Plastic Strength	f_y	14.000	MPa

In the orthotropic case, the problem is described by the following set of parameters.

Material	Orthotropic Elastic-Plastic	Modulus of Elasticity	$E_x = E_y = E_z$	11000.000	MPa
		Shear Modulus	$G_{yz} = G_{xz} = G_{xy}$	5500.000	MPa
		Poisson's Ratio	$\nu_{yz} = \nu_{xz} = \nu_{xy}$	0.000	–
		Tensile Plastic Strength	$f_{t,x} = f_{t,y}$	16.000	MPa
			$f_{t,z}$	14.000	MPa
		Compressive Plastic Strength	$f_{c,x} = f_{c,y}$	21.000	MPa
			$f_{c,z}$	24.000	MPa
		Shear Plastic Strength	$f_{v,yz} = f_{v,xz}$	2.700	MPa
			$f_{v,xy}$	1.000	MPa

In both isotropic and orthotropic case the upper block remains in the elastic state.

Geometry	Block	Width	w	0.050	m
		Depth	d	0.050	m
		Height	h	2.000	m
Load		Pressure	p	32.000	MPa

Small deformations are considered and the self-weight is neglected in this example. Determine the maximum deflection $u^1_{z,\max}$ and residual deflection $u^2_{z,\max}$. The following load case sequence is applied:

1. $p_1 = p = 32$ MPa (elastic-plastic loading)
2. $p_2 = 0$ MPa (unloading)

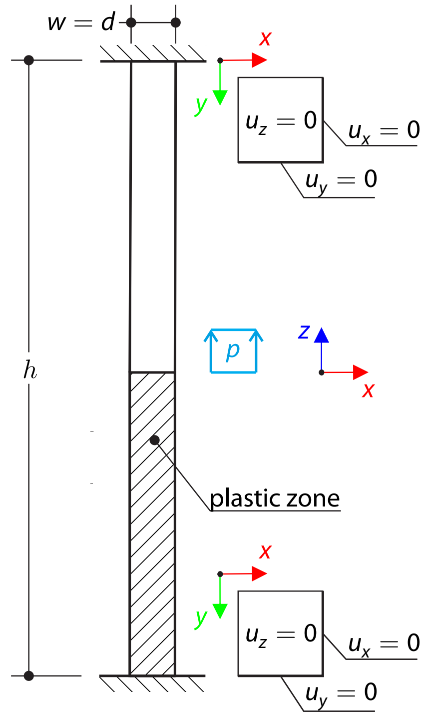

Figure 1: Problem sketch

Analytical Solution

Linear Analysis

Considering linear analysis (only elasticity) the maximum deflection of the structure can be calculated as follows:

$$u_{z,\max,\text{el}} = \left(\frac{p_1}{2E}\right)\frac{h}{2} \approx 1.455 \text{ mm} \tag{8 – 1}$$

Isotropic Case

Let us derive the analytical solution in the isotropic case, the orthotropic case is identical (with different notation only). During the first load case compressed part (upper block) remains elastic, whereas tensioned part (lower block) becomes plastic. The plastic part of the block transfers only stress $f_{t,z} = 14$ MPa, therefore the elastic part of the block is loaded by the remaining stress. The maximum displacement $u^1_{z,\max}$ is the following

$$u^1_{z,\max} = \varepsilon \frac{h}{2} = \left(\frac{p_1 - f_y}{E}\right) \frac{h}{2} \approx 1.636 \text{ mm} \qquad (8-2)$$

The maximum elastic displacement $u_{z,\max,el}$ is calculated in equation **(8 – 1)**.

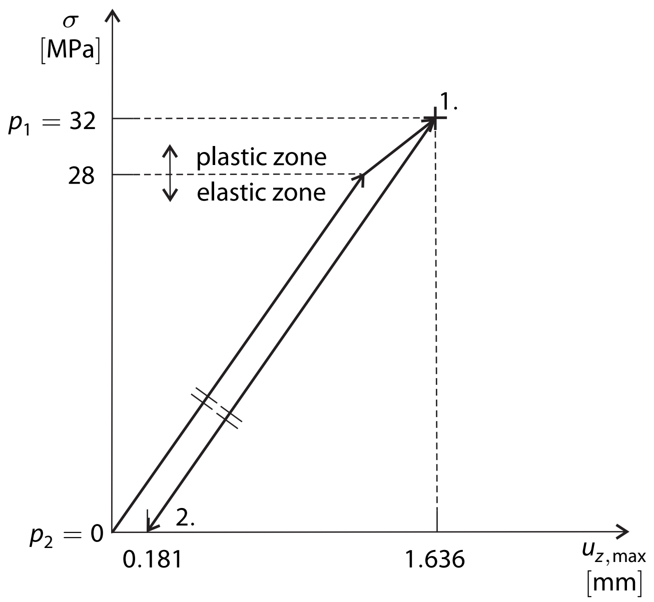

Figure 2: Load cases

The residual displacement after the second load case is calculated according to the **Figure 2**:

$$u^2_{z,\max} = u^1_{z,\max} - u_{z,\max,el} \approx 0.182 \text{ mm} \qquad (8-3)$$

Remark: In case of orthotropic material, the whole construction remains stable due to the material property $f_{c,z} > f_{t,z}$. The calculation is identical with the isotropic case.

RFEM 5 Settings

- Modeled in RFEM 5.16.01
- The element size is $l_{FE} = 0.100$ m
- Geometrically linear analysis is considered
- The number of increments is 5

Results

Structure File	Program	Entity	Material model	Description
0008.01	RF-LOAD-HIS-TORY	Solid	Orthotropic Plastic 3D	Tsai-Wu Hypothesis
0008.02	RF-LOAD-HIS-TORY	Plate	Orthotropic Plastic 2D	Tsai-Wu Hypothesis
0008.03	RF-LOAD-HIS-TORY	Plate	Isotropic Plastic 2D/3D	von Mises Hypothesis
0008.04	RF-LOAD-HIS-TORY	Solid	Isotropic Plastic 2D/3D	von Mises Hypothesis
0008.05	RF-LOAD-HIS-TORY	Member	Isotropic Nonlinear Elastic 1D	-
0008.06	RF-LOAD-HIS-TORY	Plate	Isotropic Nonlinear Elastic 2D/3D	Drucker-Prager Hypothesis
0008.07	RF-LOAD-HIS-TORY	Solid	Isotropic Nonlinear Elastic 2D/3D	Drucker-Prager Hypothesis
0008.08	RF-LOAD-HIS-TORY	Plate	Isotropic Nonlinear Elastic 2D/3D	Mohr-Coulomb Hypothesis
0008.09	RF-LOAD-HIS-TORY	Solid	Isotropic Nonlinear Elastic 2D/3D	Mohr-Coulomb Hypothesis
0008.10	RF-LOAD-HIS-TORY	Member	Isotropic Nonlinear Elastic 1D	Truss Member type
0008.11	RF-LOAD-HIS-TORY	Plate	Isotropic Nonlinear Elastic 2D/3D	von Mises Hypothesis
0008.12	RF-LOAD-HIS-TORY	Plate	Isotropic Nonlinear Elastic 2D/3D	Tresca Hypothesis
0008.13	RFEM 5	Member	Isotropic Nonlinear Elastic 1D	Temperature load

Structure File	Program	Entity	Material model	Description
0008.14	RFEM 5	Member	Isotropic Nonlinear Elastic 1D	Curved beam
0008.15	RF-LOAD-HIS-TORY	Member	Isotropic Plastic 1D	-
0008.16	RF-LOAD-HIS-TORY	Plate	Isotropic Plastic 2D/3D	Drucker-Prager Hypothesis
0008.17	RF-LOAD-HIS-TORY	Solid	Isotropic Plastic 2D/3D	Drucker-Prager Hypothesis
0008.18	RF-LOAD-HIS-TORY	Plate	Isotropic Plastic 2D/3D	Mohr-Coulomb Hypothesis
0008.19	RF-LOAD-HIS-TORY	Solid	Isotropic Plastic 2D/3D	Mohr-Coulomb Hypothesis

Load case 1

Model	Analytical Solution	RFEM 5 / RF-LOAD-HISTORY	
	$u^1_{z,max}$ [mm]	$u^1_{z,max}$ [mm]	Ratio [-]
Orthotropic Plastic 3D	1.636	1.637	1.001
Orthotropic Plastic 2D		1.637	1.001
Isotropic Plastic 2D/3D, Plate, von Mises		1.635	0.999
Isotropic Plastic 2D/3D, Solid, von Mises		1.633	0.980
Isotropic Nonlinear Elastic 1D		1.636	1.000
Isotropic Nonlinear Elastic 2D/3D, Plate, Drucker-Prager		1.636	1.000
Isotropic Nonlinear Elastic 2D/3D, Solid, Drucker-Prager		1.636	1.000
Isotropic Nonlinear Elastic 2D/3D, Plate, Mohr-Coulomb		1.636	1.000
Isotropic Nonlinear Elastic 2D/3D, Solid, Mohr-Coulomb		1.636	1.000
Isotropic Nonlinear Elastic 1D, Truss		1.636	1.000
Isotropic Nonlinear Elastic 2D/3D, Plate, von Mises		1.636	1.000
Isotropic Nonlinear Elastic 2D/3D, Plate, Tresca		1.636	1.000
Isotropic Nonlinear Elastic 1D, Tempera-ture Load		1.636	1.000
Isotropic Nonlinear Elastic 1D, Curved beam		1.637	1.001

Model	Analytical Solution	RFEM 5 / RF-LOAD-HISTORY	
	$u^1_{z,max}$ [mm]	$u^1_{z,max}$ [mm]	Ratio [-]
Isotropic Plastic 1D	1.636	1.636	1.000
Isotropic Plastic 2D/3D, Plate, Drucker-Prager		1.636	1.000
Isotropic Plastic 2D/3D, Solid, Drucker-Prager		1.636	1.000
Isotropic Plastic 2D/3D, Plate, Mohr-Coulomb		1.636	1.000
Isotropic Plastic 2D/3D, Solid, Mohr-Coulomb		1.636	1.000

Load case 2

Model	Analytical Solution	RF-LOAD-HISTORY	
	$u^2_{z,max}$ [mm]	$u^2_{z,max}$ [mm]	Ratio [-]
Orthotropic Plastic 3D	0.182	0.182	1.000
Orthotropic Plastic 2D		0.182	1.000
Isotropic Plastic 2D/3D, Plate, von Mises		0.181	0.995
Isotropic Plastic 2D/3D, Solid, von Mises		0.178	0.978
Isotropic Plastic 1D		0.182	1.000
Isotropic Plastic 2D/3D, Plate, Drucker-Prager		0.182	1.000
Isotropic Plastic 2D/3D, Solid, Drucker-Prager		0.182	1.000
Isotropic Plastic 2D/3D, Plate, Mohr-Coulomb		0.182	1.000
Isotropic Plastic 2D/3D, Solid, Mohr-Coulomb		0.182	1.000

Remark: In case of nonlinear elastic material models, the residual deflection $u^2_{z,max}$ has to be zero, because these models do not allow plastic behaviour. The nonlinear elastic material models loads and unloads along the same curve. Due to the clarity, the zero residual deflections are not introduced in the result table.