Program: RFEM 5, RFEM 6

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Temperature Dependency, Plate

Verification Example: 0022 – Circle Surface with Thermal Loading

0022 – Circle Surface with Thermal Loading

Description

A simply supported circle plate is subjected to the uniform pressure p, uniform temperature T_c and differential temperature ΔT. The reference temperature t_0 is 20°C. Assuming only small deformations and neglecting plate's self-weight, determine it's maximum deflection $u_{z,\max}$ and maximum radial moment $m_{r,\max}$.

Material	Steel	Modulus of Elasticity	$E_{20°C}$	210.000	GPa
			$E_{40°C}$	190.000	GPa
		Poisson's Ratio	ν	0.300	–
		Coefficient of Thermal Expansion	α	1.2×10^{-5}	1/K
Geometry	Plate	Radius	r	1.000	m
		Thickness	t	0.040	m
Loading	Pressure	Uniform	p	0.100	MPa
	Temperature	Uniform	T_c	10.000	°C
		Difference	ΔT	−50.000	°C

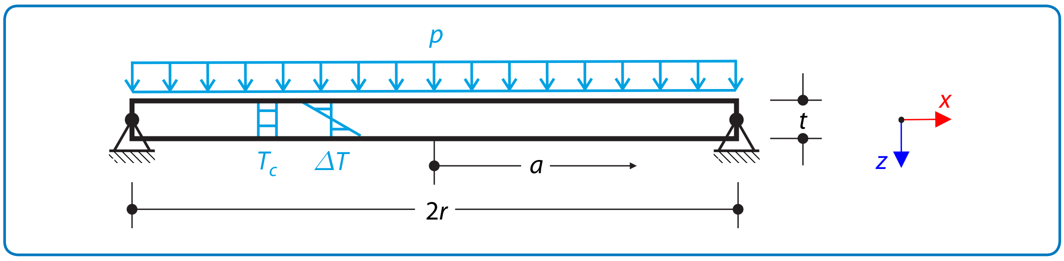

Figure 1: Problem sketch

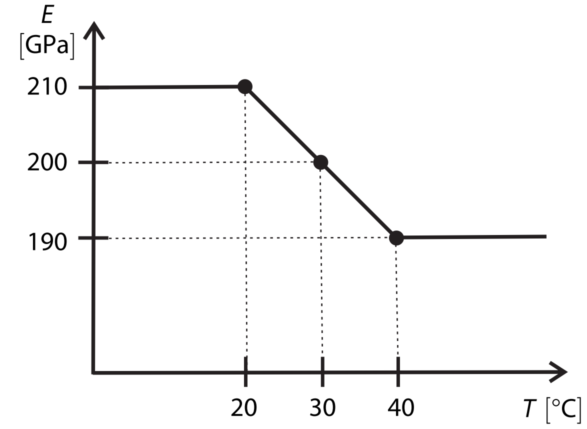

Figure 2: Temperature chart

Analytical Solution

The problem can be analytically solved by the integration of the bending equation for circle plates [1] and the final expressions for the deflection $u_{z,\mathrm{max}}$ and moment m_r are can be derived from the Kirchhoff's equations for plates.

The maximum deflection of the plate consists of deflection due to the temperature and pressure:

$$u_{z,\mathrm{max}} = u_{z,\mathrm{temp}} + u_{z,\mathrm{press}} \tag{22 – 1}$$

The deflection due to the temperature can be derived from the equation for the plate's curvature:

$$\kappa = \frac{\varphi}{a} = \frac{12}{Eh^3} m_\mathrm{T} \tag{22 – 2}$$

where a is the distance from the center, φ is the plate's rotation:

$$\varphi = \frac{\mathrm{d}u_{z,\mathrm{temperature}}}{\mathrm{d}a} \tag{22 – 3}$$

and m_T is the moment in the plate caused by the temperature difference and can be evaluated by the following formula:

$$m_\mathrm{T} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \alpha E T(z) z \, \mathrm{d}z \tag{22 – 4}$$

where $T(z)$ is the function of the temperature change through the plate thickness:

$$T(z) = \frac{z}{t} \Delta T \tag{22 – 5}$$

Combining the equation **(22 – 3)** with the equation **(22 – 2)**, formula for the deflection $u_{z,\mathrm{temp}}$ can be obtained:

$$u_{z,\text{temp}} = \int \frac{12}{Eh^3} m_\text{T} a \, \text{d}a \tag{22 – 6}$$

Integrating the equation **(22 – 6)**, while assuming that the deflection at the plate's edge is equal to zero ($u_{z,\text{temp}}(r) = 0$), the final expression for the deflection due to the temperature can be obtained:

$$u_{z,\text{temp}} = \frac{\alpha \Delta T}{2t} \left(a^2 - r^2 \right) \tag{22 – 7}$$

The deflection due to the pressure can be expressed as:

$$u_{z,\text{press}} = \frac{p_0 a^4}{64D} - \frac{p_0 r^4}{64D} + \frac{p_0 r^4 (3 + \nu)}{32(D + \nu D)} - \frac{p_0 a^2 r^2 (3 + \nu)}{32(D + \nu D)} \tag{22 – 8}$$

where D is plate rigidity:

$$D = \frac{E_\text{final} t^3}{12(1 - \nu^2)} \tag{22 – 9}$$

where E_final is modulus of elasticity for the final temperature defined as a sum of the reference temperature and uniform temperature load and can be deducted from the **Figure 2**:

$$T_\text{final} = t_0 + T_\text{c} = 20 + 10 = 30°C \tag{22 – 10}$$

The maximum radial moment in the center of the plate ($a = 0$ m) can be obtained from the deflection due to the pressure:

$$m_{\text{r,max}} = D \left(\frac{\text{d}^2 u_{z,\text{press}}}{\text{d}a^2} + \frac{\nu}{a} \frac{\text{d}u_{z,\text{press}}}{\text{d}a} \right) = \frac{p_0 r^2 (3 + \nu)}{16} \tag{22 – 11}$$

RFEM Settings

- Modeled in version RFEM 5.26 and RFEM 6.01
- The element size is $l_\text{FE} = 0.025$ m
- Geometrically linear analysis is considered
- The Mindlin plate theory is used
- Isotropic thermal-elastic material model is used

Results

Structure File	Program	Entity
0022.01	RFEM 5, RFEM 6	Plate

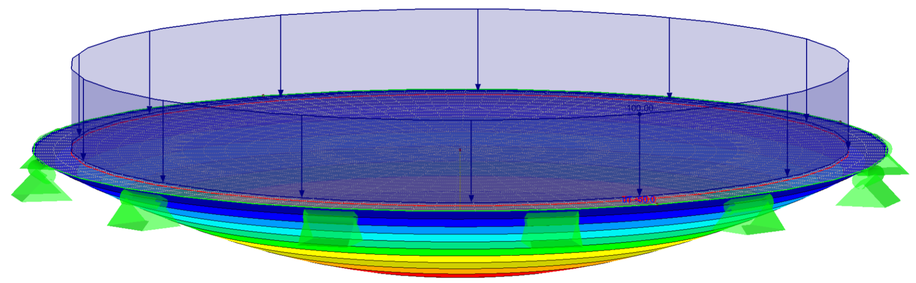

Figure 3: RFEM 5 Results

As can be seen in the tables below, an excellent consensus of analytical solutions for deflection $u_{z,\mathrm{max}}$ and radial moment $m_{\mathrm{r,max}}$ with RFEM 5 outputs was achieved.

Analytical Solution	RFEM 5		RFEM 6	
$u_{z,\mathrm{max}}$ [mm]	$u_{z,\mathrm{max}}$ [mm]	Ratio [-]	$u_{z,\mathrm{max}}$ [mm]	Ratio [-]
12.935	12.943	1.001	12.946	1.001

Analytical Solution	RFEM 5		RFEM 6	
$m_{\mathrm{r,max}}$ [kNm/m]	$m_{\mathrm{r,max}}$ [kNm/m]	Ratio [-]	$m_{\mathrm{r,max}}$ [kNm/m]	Ratio [-]
20.625	20.681	1.003	20.715	1.004

References

[1] SZILARD, R. *Theories and Application of Plate Analysis: Classical Numerical and Engineering Method*. Hoboken, New Jersey, 2004.