Program: RFEM 5, RFEM 6

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Plate

Verification Example: 0063 – Centrifugal Force Loading

0063 – Centrifugal Force Loading

Description

A compact disc (CD) rotates at a speed 10 000 rpm. Therefore, it is subjected to the centrifugal force. The problem is modeled as a quarter model, see **Figure 1**. The problem is described by the following set of parameters.

Material	Polycarbonate	Modulus of Elasticity	E	850.000	MPa
		Poisson's Ratio	ν	0.300	–
		Density	ρ	1190.000	kg/m^3
Geometry		Inner radius	r_1	7.500	mm
		Outer radius	r_2	60.000	mm
		Thickness	t	1.2	mm
Load		Rotary Motion	ω	1047.200	rad/s

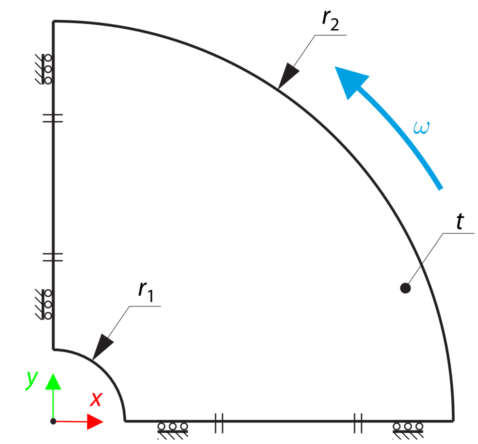

Figure 1: Problem Sketch

Determine the tangential stress σ_t on the inner and outer diameter at first, then determine the radial deflection of the outer radius $u_r(r_2)$. The self-weight is neglected.

Analytical Solution

The tangential stress σ_t and radial stress σ_r on a thin rotating disc is defined as follows:

$$\sigma_t(r) = C_1 + \frac{C_2}{r^2} - \frac{1+3\nu}{8}\rho\omega^2 r^2 \qquad (63-1)$$

$$\sigma_r(r) = C_1 + \frac{C_2}{r^2} - \frac{3+\nu}{8}\rho\omega^2 r^2 \qquad (63-2)$$

where C_1 and C_2 are real constants, which can be obtained from the boundary condition of zero radial stress σ_r both on the inner and outer diameter.

$$\sigma_r(r_1) = 0 \tag{63 – 3}$$

$$\sigma_r(r_2) = 0 \tag{63 – 4}$$

Therefore, constants C_1 and C_2 are equal to:

$$C_1 = \frac{3+\nu}{8}\rho\omega^2(r_1^2 + r_2^2) \tag{63 – 5}$$

$$C_2 = \frac{3+\nu}{8}\rho\omega^2 r_1^2 r_2^2 \tag{63 – 6}$$

The tangential stress σ_t on the inner and outer diameter is calculated according to formula **(63 – 1)**.

$$\sigma_t(r_1) = 3.889 \text{ MPa} \tag{63 – 7}$$

$$\sigma_t(r_2) = 0.883 \text{ MPa} \tag{63 – 8}$$

The radial deflection of the outer radius can be calculated using the Hooke's Law. Note that the radial stress on the outer radius is equal to zero.

$$u_r(r_2) = \frac{r_2}{E}\left[\sigma_t(r_2) - \nu\sigma_r(r_2)\right] = 0.0623 \text{ mm} \tag{63 – 9}$$

RFEM Settings

- Modeled in RFEM 5.06 and RFEM 6.01
- The element size is $l_{FE} = 0.001$ m
- The number of increments is 5
- Isotropic linear elastic material model is used
- Kirchhoff plate bending theory is used

Results

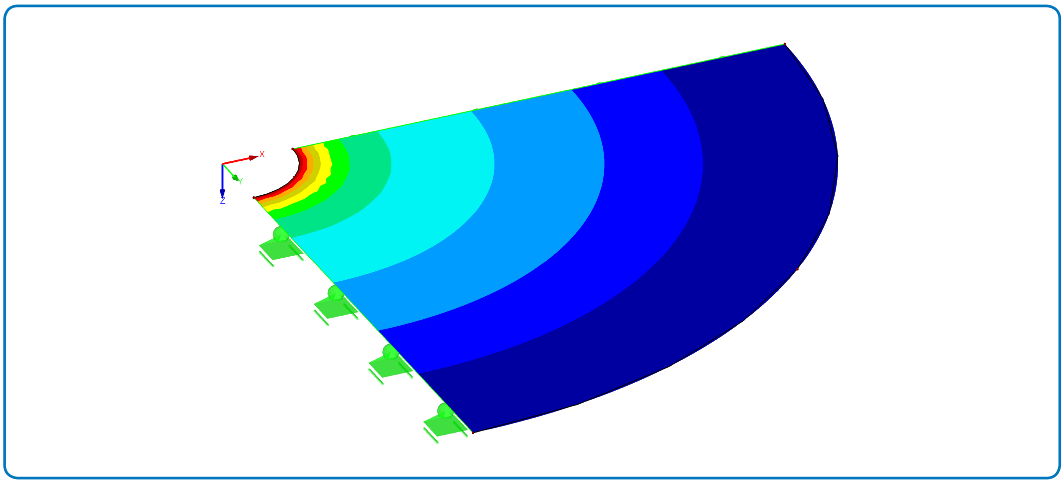

Figure 2: Results in RFEM - von Mises stress

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Structure Files	Program
0063.01	RFEM 5, RFEM 6

Quantity	Analytical Solution	RFEM 5	Ratio	RFEM 6	Ratio
$\sigma_t(r_1)$ [MPa]	3.889	3.891	1.001	3.891	1.001
$\sigma_t(r_2)$ [MPa]	0.883	0.882	0.999	0.882	0.999
$u_r(r_2)$ [mm]	0.0623	0.0623	1.000	0.0623	1.000