0075 – Combined Loading - Reduced Stress

Description

A cantilever with circular cross-section is loaded by a concentrated bending force and torque. The aim of this verification example is to compare the reduced stress according to the von Mises and Tresca theories.

Material	Modulus of Elasticity	E	210000.000	MPa
	Poisson's Ratio	ν	0.296	–
Geometry	Diameter	d	50.000	mm
	Length	L	1000.000	mm
Load	Bending Force	F	1000.000	N
	Torque	M	1.000×10^6	Nmm

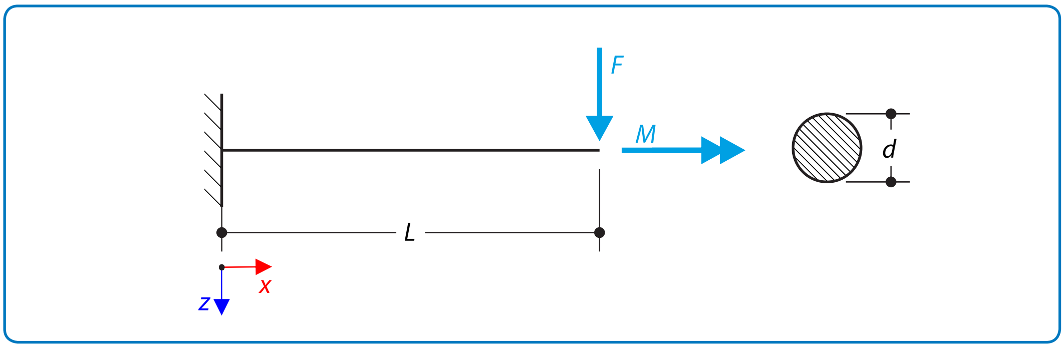

Figure 1: Problem Sketch

While neglecting self-weight, determine the reduced stress according to the von Mises and Tresca theory $\sigma_{\text{Mises}}(L/2)$, $\sigma_{\text{Tresca}}(L/2)$ at the mid-point of the cantilever.

Analytical Solution

The resultant total stress at the mid-point of the cantilever is decomposed into three components, namely the bending and shear stresses σ_{b}, τ_z due to the concentrated force F, and the shear stress τ due to the torque M.

$$\sigma_{\text{b}}(L/2) = \frac{FL}{2S_y}, \qquad \tau_z(L/2) = \frac{F}{A}, \qquad \tau(L/2) = \frac{M}{S_t} \tag{75 – 1}$$

where S_y and S_t are the elastic and torsional section moduli of the cross-section, and A is the area of the cross-section

$$S_y = \frac{\pi d^3}{32}, \qquad S_t = \frac{\pi d^3}{16}, \qquad A = \frac{\pi d^2}{4} \tag{75 – 2}$$

Taking into account all above mentioned stress components, the reduced stress is computed according to the von Mises and Tresca theories

$$\sigma_{\text{Mises}}(L/2) = \sqrt{\sigma^2 + 3\,(\tau + \tau_z)^2} \approx 82.252 \text{ MPa} \tag{75 – 3}$$

$$\sigma_{\text{Tresca}}(L/2) = \sqrt{\sigma^2 + 4\,(\tau + \tau_z)^2} \approx 92.018 \text{ MPa} \tag{75 – 4}$$

RFEM 5 Settings

- Modeled in RFEM 5.07.05
- The element size is $l_{\text{FE}} = 0.005$ m
- The number of increments is 10
- Isotropic linear elastic model is used
- Shear stiffness of members is activated

Results

Structure Files	Entity
0075.01	Member
0075.02	Solid

Quantity [MPa]	Analytical Solution	RFEM 5 Member	Ratio	RFEM 5 Solid	Ratio
$\sigma_{\text{Mises}}(L/2)$	82.252	81.693	0.993	82.554	1.004
$\sigma_{\text{Tresca}}(L/2)$	92.018	91.344	0.993	92.273	1.003

Remark

When using member entity, the cantilever is modeled at half the length to obtain results equivalent to the solid model. For member models, the results are read in the fixed node. For solid models, the results are read from the section in the half of the cantilever.