Program:	RFEM 5
Category:	Isotropic Linear Elasticity, Geometrically Linear Analysis, Shell
Verification Example:	0085 – Thin-Walled Conical Vessel with Hydrostatic Pressure

0085 – Thin-Walled Conical Vessel with Hydrostatic Pressure

Description

A thin-walled conical vessel of height h and peak angle 2φ is filled with water. Thus, it is loaded by the hydrostatic pressure according to **Figure 1**. While neglecting self-weight, determine the stresses σ_1 and σ_2 at the test point at height $h_0 = 1.000$ m.

Material	Modulus of Elasticity	E	210000.000	MPa
	Poisson's Ratio	ν	0.296	–
Geometry	Vessel Height	h	2.000	m
	Shell Thickness	t	1.000	mm
	Vessel Angle	φ	$\pi/6$	rad
Load	Water Specific Weight	γ	9810.000	N/mm^3

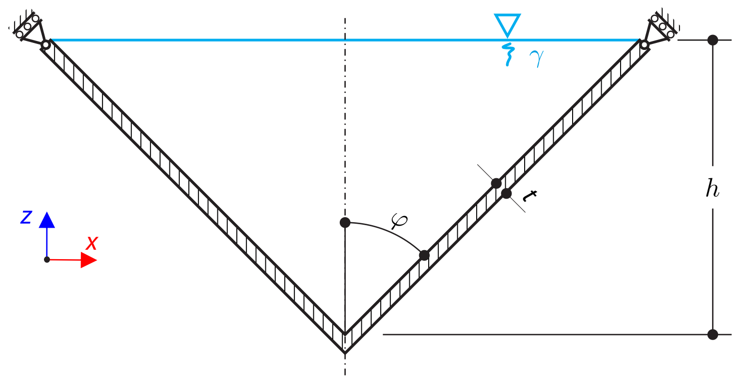

Figure 1: Problem Sketch

Analytical Solution

The analytical solution is based on the theory of thin-walled vessels. This theory was introduced in Verification Example 0084, see [1]. The stress state of the thin-walled vessel is described by the Laplace equation

$$\frac{\sigma_1}{R_1} + \frac{\sigma_2}{R_2} = \frac{p}{t}, \qquad (85-1)$$

where σ_1, σ_2 are stresses in surface line and circumferential direction respectively and R_1, R_2 are the radii in the corresponding directions. The mentioned stresses correspond to the principal stresses. The pressure p is in this case equal to the hydrostatic pressure[1]

[1] The specific wheight γ is equal to $\gamma = \rho g$, where ρ is the fluid density and g is the gravitational acceleration.

$$p = h\rho g = \gamma h. \tag{85 – 2}$$

The radius R_1 for the conical vessel is equal to $R_1 \approx \infty$. The radius R_2 can be expressed, in accordance with the sketch in **Figure 2**, considering $r = z \tan\varphi$

$$R_2 = \frac{r}{\cos\varphi} = z\frac{\sin\varphi}{\cos^2\varphi}. \tag{85 – 3}$$

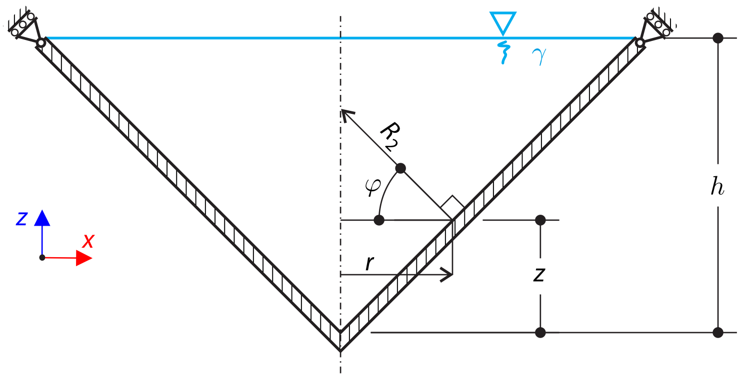

Figure 2: Sketch of used dimensions of conical vessel

The pressure in the depth $h - z$ is equal to

$$p(z) = \gamma(h - z). \tag{85 – 4}$$

Substituting into the the equation **(85 – 1)**, circumferential stress σ_2 can be obtained

$$\sigma_2 = \frac{\gamma(h - z)z\sin\varphi}{t\cos^2\varphi}. \tag{85 – 5}$$

An additional equation has to be defined to obtain the remaining stress σ_1. The internal and external forces have to be equal. Furthermore, the external force Q due to the hydrostatic pressure is equal to the gravity force caused by the height of the water column

$$Q = \sigma_1 2\pi rt\cos\varphi, \tag{85 – 6}$$

$$Q = \gamma\left[\pi r^2(h - z) + \frac{1}{3}\pi r^2 z\right]. \tag{85 – 7}$$

The desired stress σ_1 can then be determined using **(85 – 6)** - **(85 – 7)**

$$\sigma_1 = \frac{\gamma z\sin\varphi}{2t\cos^2\varphi}\left(h - \frac{2}{3}z\right). \tag{85 – 8}$$

For the test point at height $z = 1.000$ m, the above mentioned quantities can be calculated

$$\sigma_1 \approx 9.249 \text{ MPa}, \tag{85 – 9}$$

$$\sigma_2 \approx 13.873 \text{ MPa}. \tag{85 – 10}$$

RFEM 5 Settings

- Modeled in RFEM 5.12.02
- Element size is $l_{FE} = 0.025$ m
- The number of increments is 10
- Isotropic linear elastic material is used
- Kirchhoff plate bending theory is used

Note that the hydrostatic pressure can be conveniently modeled by means of Free Rectangular Load in RFEM 5 according to **Figure 3**. The pressure at the top edge ($z = 2.000$ m) is equal to $p_1 = 0.000$ N/m^2 and the pressure at the bottom of the vessel ($z = 0.000$ m) is defined by the equation for the hydrostatic pressure **(85 – 2)** and $p_2 = -19620.000$ N/m^2.

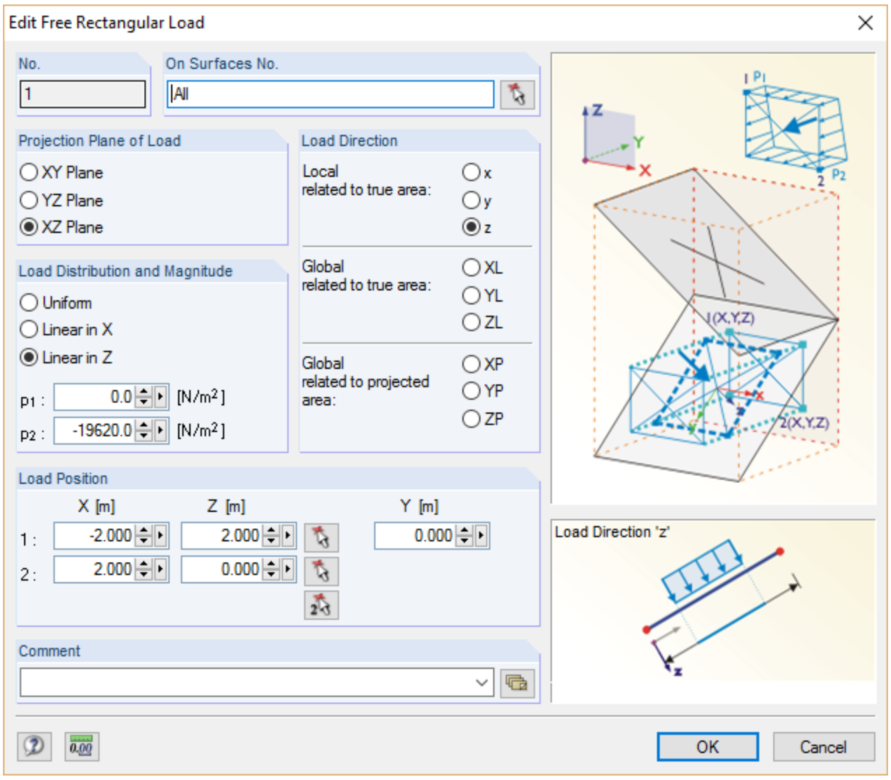

Figure 3: Free Rectangular Load definition in RFEM 5

Results

Structure File	Program
0085.01	RFEM 5

Remark: The stresses σ_1 and σ_2 are evaluated at the middle surface of the conical vessel. The corresponding stresses in RFEM 5 are $\sigma_{2,m}$ and $\sigma_{1,m}$, respectively.

Quantity	Analytical Solution	RFEM 5	Ratio
σ_1 [MPa]	9.249	9.264	1.002
σ_2 [MPa]	13.873	13.982	1.008

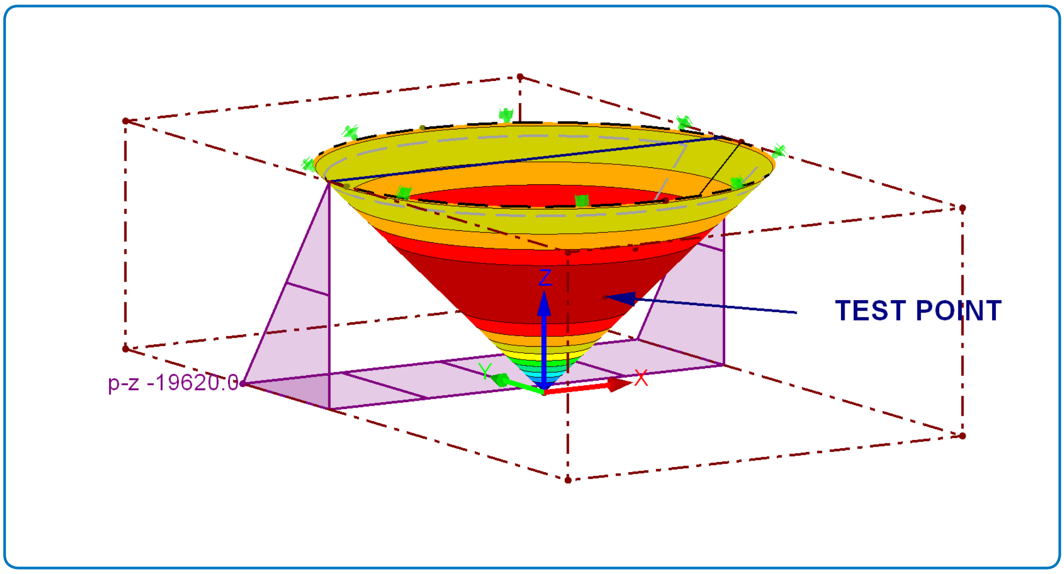

Figure 4: Von Mises stress distribution, Free Rectangular Load for the hydrostatic pressure and the test point location in RFEM 5

References

[1] DLUBAL SOFTWARE GMBH, *Verification Example 0084 – Thin-walled Spherical Vessel*. 2017.