Program: RFEM 5

Category: Orthotropic Linear Elasticity, Geometrically Linear Analysis, Plate

Verification Example: 0091 – Thin Rectangular Orthotropic Plate Under Uniform Load

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Description

Thin rectangular orthotropic plate is simply supported and loaded by the uniformly distributed pressure p according to **Figure 1**. The directions of axis x and y coincide with the principal directions. While neglecting self-weight, determine the maximum deflection $u_{z,\max}$ of the plate.

Material	Orthotropic	Modulus of Elasticity	E_x	10000.000	MPa
			E_y	670.000	MPa
		Poisson's Ratio	ν_{xy}	0.200	–
		Shear Modulus	G_{xy}	620.000	MPa
Geometry		Length	a	2.000	m
		Width	b	1.000	m
		Thickness	t	0.010	m
Load		Pressure	p	100.000	Pa

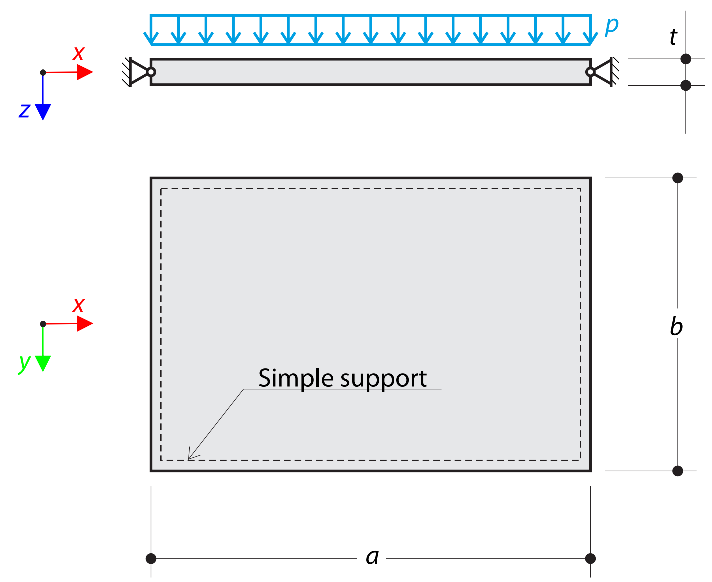

Figure 1: Problem Sketch

Analytical Solution

The deflection $u_z(x,y)$ of rectangular orthotropic plate with coincident axis of geometry and material axis is described by the partial differential equation [1]

$$D_{xx}\frac{\partial^4}{\partial x^4}u_z + 2H\frac{\partial^4}{\partial x^2 \partial y^2}u_z + D_{yy}\frac{\partial^4}{\partial y^4}u_z = p, \qquad (91-1)$$

where p is the surface loading. Stiffness constants for orthotropic plate D_{xx}, D_{yy} and $H = D_{xx}\nu_{yx} + 2D_{xy}$ are equal to

$$D_{xx} = \frac{E_x t^3}{12(1 - \nu_{xy}\nu_{yx})}, \qquad (91-2)$$

$$D_{yy} = \frac{E_y t^3}{12(1 - \nu_{xy}\nu_{yx})}, \qquad (91-3)$$

$$D_{xy} = \frac{G_{xy} t^3}{12}. \qquad (91-4)$$

The Poisson's ration ν_{yx} is defined as

$$\nu_{yx} = \frac{E_y \nu_{xy}}{E_x}. \qquad (91-5)$$

To solve the differential equation **(91 – 1)** the Navier method (see [2]) is used. Considering the simply supported rectangular plate, the deflection can be written in the form of double Fourier series

$$u_z(x,z) = \sum_m \sum_n W_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right), \qquad m = 1,2,3,\ldots; n = 1,2,3,\ldots \qquad (91-6)$$

In order to determine the coefficients W_{mn},the uniform load function $p(x,y)$ is also expanded into double Fourier series

$$p = \sum_m \sum_n a_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right), \qquad (91-7)$$

where the Fourier coefficient a_{mn} is equal to

$$a_{mn} = \frac{4}{ab} \int_0^a \int_0^b p \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \mathrm{d}x\mathrm{d}y = \qquad (91-8)$$
$$= \frac{4p}{\pi^2 mn} \left(cos(\pi m) - 1\right)\left(cos(\pi n) - 1\right)$$

The coefficients W_{mn} are obtained by substituting **(91 – 6)** and **(91 – 7)** into **(91 – 1)**

$$W_{mn} = \frac{a_{mn}}{\pi^4} \frac{1}{D_{xx}\left(\frac{m}{a}\right)^4 + 2H\left(\frac{mn}{ab}\right)^2 + D_{yy}\left(\frac{n}{b}\right)^4}. \qquad (91-9)$$

The deflection function $u_z(x,y)$ then results in

$$u_z = \frac{4p}{\pi^6} \sum_m \sum_n \frac{\left(cos(\pi m) - 1\right)\left(cos(\pi n) - 1\right)}{mn\left(D_{xx}\left(\frac{m}{a}\right)^4 + 2H\left(\frac{mn}{ab}\right)^2 + D_{yy}\left(\frac{n}{b}\right)^4\right)} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right). \qquad (91-10)$$

The maximum deflection $u_{z,\max}$ of the plate is located at the center $x = \frac{a}{2}, y = \frac{b}{2}$ and approximately is equal to

$$u_{z,\max} \approx 9.860 \text{ mm.} \tag{91 – 11}$$

RFEM 5 Settings

- Modeled in RFEM 5.13.01
- Element size is $l_{FE} = 0.050$ m
- The number of increments is 10
- Orthotropic linear elastic material is used

Results

Structure File	Plate bending theory	Finite Element Shape
0091.01	Kirchhoff	Quadrangle
0091.02	Kirchhoff	Triangle
0091.03	Mindlin	Quadrangle
0091.04	Mindlin	Triangle

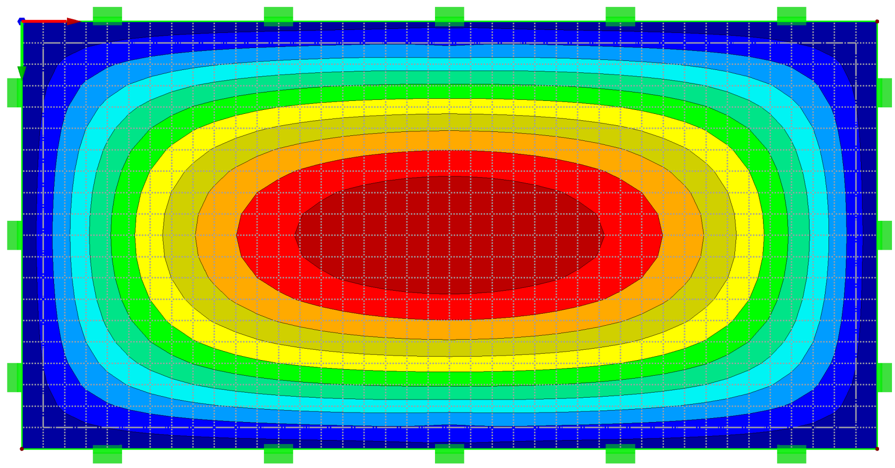

Figure 2: RFEM 5 results – deflection u_z

Test Variant	Theory	RFEM 5	
	$u_{z,\max}$ [mm]	$u_{z,\max}$ [mm]	Ratio [-]
Kirchhoff, Quadrangle	9.860	9.850	0.999
Kirchhoff, Triangle		9.865	1.001
Mindlin, Quadrangle		9.845	0.998
Mindlin, Triangle		9.792	0.993

References

[1] TIMOSHENKO, S. *Theory of Plates and Shells*. McGraw-Hill Book Company, 1940.

[2] LEKHNITSKIY, S. *Anisotropic Plates. Translated from the Second Russian Edition by S.W. Tsai and T. Cheron*. 1968.