0100 – Single Mass System Oscillation

Description

A single mass system is subjected to the loading force F according to the **Figure 1**. The problem is described by the following parameters.

System Properties	Spring	Stiffness	k	200.000	N/m
		Length	L	1000.000	m
		Damping Parameter	b	0.500	Ns/m
	Mass	Weight	m	0.500	kg
Load		Force	F	200.000	N

Determine the deflection u_y for two values of the damping parameter[1] ($b = 0.000$ Ns/m and $b = 0.500$ Ns/m) in test time $t_1 = \frac{\pi}{20}$ s and $t_2 = \frac{3\pi}{20}$ s.

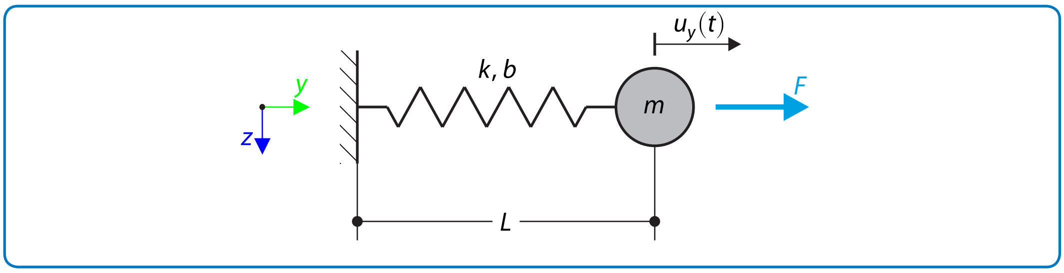

Figure 1: Problem sketch

Analytical Solution

A single mass system is described by the second order differential equation.

$$m\frac{\mathrm{d}^2 u_y(t)}{\mathrm{d}t^2} + b\frac{\mathrm{d}u_y(t)}{\mathrm{d}t} + ku_y(t) = F \qquad (100-1)$$

The homogeneous solution of the differential equation is found using the characteristic equation.

$$m\lambda^2 + b\lambda + k = 0 \qquad (100-2)$$

The roots of this equation yield

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m} \qquad (100-3)$$

[1] Damping matrix is defined in RF-DYNAM by means of Rayleigh damping $\boldsymbol{B} = \alpha\boldsymbol{M} + \beta\boldsymbol{K}$, where α and β are damping coefficients.

The homogeneous solution can be then determined.

$$u_{yH}(t) = C_1 e^{-\frac{\left(b+\sqrt{b^2-4mk}\right)t}{2m}} + C_2 e^{\frac{\left(-b+\sqrt{b^2-4mk}\right)t}{2m}} \tag{100 – 4}$$

Together with the particular solution the complete solution can be written.

$$u_{y}(t) = C_1 e^{-\frac{\left(b+\sqrt{b^2-4mk}\right)t}{2m}} + C_2 e^{\frac{\left(-b+\sqrt{b^2-4mk}\right)t}{2m}} + \frac{F}{k} \tag{100 – 5}$$

Where C_1 and C_2 are constants which can be determined when the following initial conditions are added.

$$u_y(0) = 0 \tag{100 – 6}$$

$$\frac{\mathrm{d}u_y(0)}{\mathrm{d}t} = 0 \tag{100 – 7}$$

The final solution then results as follows.

$$u_y(t) = -\frac{e^{-\frac{\left(b+\sqrt{b^2-4mk}\right)t}{2m}}\left(b^2-4km-b\sqrt{b^2-4km}\right)F}{2k(b^2-4km)} - \frac{e^{\frac{\left(-b+\sqrt{b^2-4mk}\right)t}{2m}}\left(b^2-4km+b\sqrt{b^2-4km}\right)F}{2k(b^2-4km)} + \frac{F}{k} \tag{100 – 8}$$

The result deflection is calculated in following test time.

$$t_1 = \frac{\pi}{20} = 0.157 \text{ s} \tag{100 – 9}$$

$$t_2 = \frac{3\pi}{20} = 0.471 \text{ s} \tag{100 – 10}$$

RFEM 5 and RSTAB 8 Settings

- Modeled in RFEM 5.08.02
- The global element size is $l_{FE} = 0.500$ m
- Spring member entity (**Figure 2**) is used

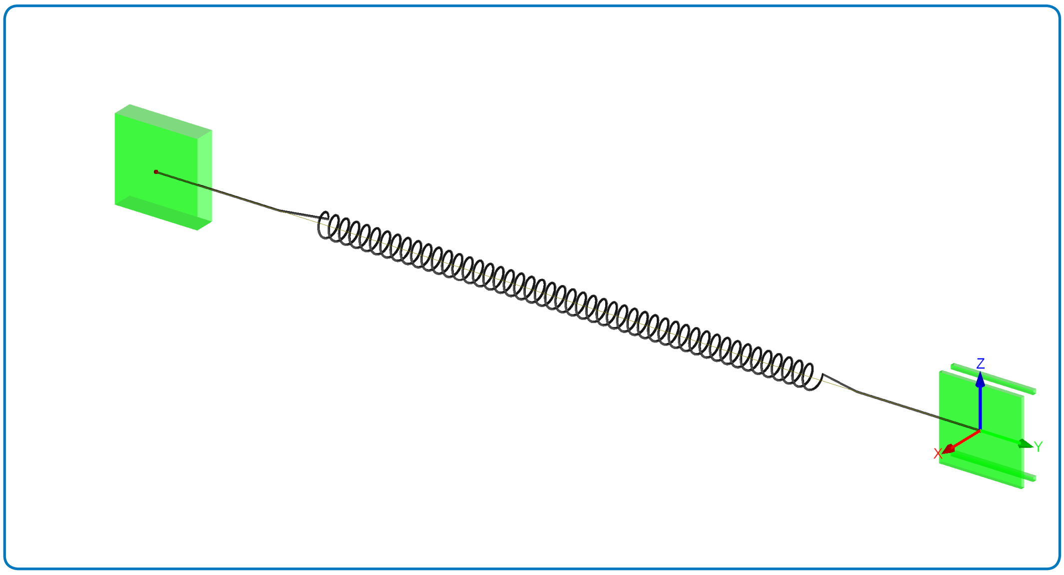

Figure 2: Spring member entity in RFEM 5

Results

Structure Files	Program	Damping Parameter	Damping Coefficients in RF-DYNAM
0100.01	RF-DYNAM Pro	$b = 0.000$ Ns/m	$\alpha = 0.000, \beta = 0.000$
0100.02	RF-DYNAM Pro	$b = 0.500$ Ns/m	$\alpha = 1.000, \beta = 0.000$

Damping Parameter $b = 0.000$ Ns/m	Analytical Solution	RF-DYNAM Solution	
	u_y [m]	u_y [m]	Ratio [-]
Test Time $t_1 = 0.157$ s	2.000	2.000	1.000
Test Time $t_2 = 0.471$ s		2.000	1.000

Damping Parameter $b = 0.500$ Ns/m	Analytical Solution	RF-DYNAM Solution	
	u_y [m]	u_y [m]	Ratio [-]
Test Time $t_1 = 0.157$ s	1.924	1.924	1.000
Test Time $t_2 = 0.471$ s	1.790	1.790	1.000