Program: RFEM 5, RSTAB 8, RF-DYNAM Pro, DYNAM Pro

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Member, Dynamics

Verification Example: 0111 – Torsional Vibrations

0111 – Torsional Vibrations

Description

A double-mass system consists of two shafts of diameters d_1, d_2 and two masses represented by the corresponding moments of inertia I_{x1}, I_{x2}, concentrated in given distance as nodal masses. The left shaft is fixed, and the right mass is free according to **Figure 1**. Neglecting the self-weight of the shafts, determine the torsional natural frequencies of the system (direction φ_x). The problem is described by the following set of parameters.

Material	Steel	Modulus of Elasticity	E	210000.000	MPa
		Shear Modulus	G	81000.000	MPa
Geometry		Cross-section Diameter	d_1	20.000	mm
		Cross-section Diameter	d_2	40.000	mm
		Mass Moment of Inertia	I_{x1}	1.000	kgm^2
		Mass Moment of Inertia	I_{x2}	0.700	kgm^2
		Shaft Length	L	0.500	m

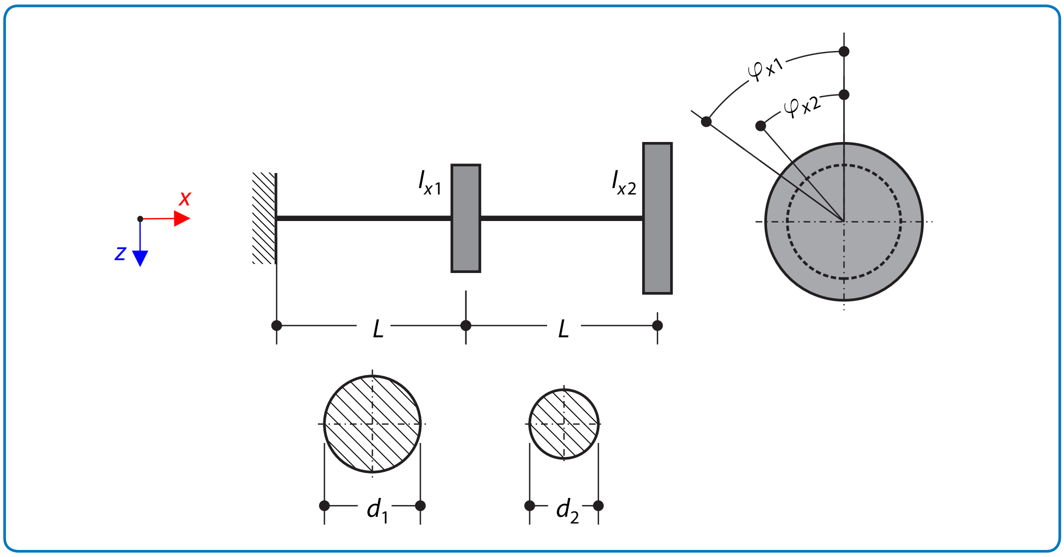

Figure 1: Problem Sketch

Analytical Solution

The system of equations for the torsional motion, assuming that $\varphi_{x1} > \varphi_{x2}$, is described as

$$\boldsymbol{M}\ddot{\varphi}_x + \boldsymbol{K}\varphi_x = 0 \tag{111 – 1}$$

$$\begin{bmatrix} I_{x1} & 0 \\ 0 & I_{x2} \end{bmatrix} \begin{bmatrix} \ddot{\varphi}_{x1} \\ \ddot{\varphi}_{x2} \end{bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \begin{bmatrix} \varphi_{x1} \\ \varphi_{x2} \end{bmatrix} = 0 \tag{111 – 2}$$

where the torsional stiffnesses k_1, k_2 are determined as

$$k_1 = \frac{\pi G d_1^4}{32L} \tag{111 – 3}$$

$$k_2 = \frac{\pi G d_2^4}{32L} \tag{111 – 4}$$

The eigenvalue problem

$$\det\left(\boldsymbol{K} - \Omega^2 \boldsymbol{M}\right) = 0 \tag{111 – 5}$$

yields the angular frequencies Ω_1 and Ω_2, whence the natural frequencies f_1, f_2 are calculated as follows

$$f_1 = \frac{\Omega_1}{2\pi} \approx 7.779 \text{ Hz} \tag{111 – 6}$$

$$f_2 = \frac{\Omega_2}{2\pi} \approx 39.615 \text{ Hz} \tag{111 – 7}$$

RFEM 5 and RSTAB 8 Settings

- Modeled in RFEM 5.09.01 and RSTAB 8.09.01
- The element size is $l_{FE} = 0.050$ m
- The number of increments is 10
- Isotropic linear elastic model is used

Results

Structure Files	Program	Member Type
0111.01	RFEM 5 - RF-DYNAM Pro	Beam
0111.02	RSTAB 8 - DYNAM Pro	Beam
0111.03	RFEM 5 - RF-DYNAM Pro	Definable Stiffness
0111.04	RSTAB 8 - DYNAM Pro	Definable Stiffness

Model	Analytical Solution	RFEM 5 / RSTAB 8	
	f_1 [Hz]	f_1 [Hz]	Ratio [-]
RFEM 5, Beam	7.779	7.779	1.000
RFEM 8, Beam		7.779	1.000
RFEM 5, Definable Stiffness		7.779	1.000
RFEM 8, Definable Stiffness		7.779	1.000

Model	Analytical Solution	RFEM 5 / RSTAB 8	
	f_2 [Hz]	f_2 [Hz]	Ratio [-]
RFEM 5, Beam	39.615	39.615	1.000
RSTAB 8, Beam		39.615	1.000
RFEM 5, Definable Stiffness		39.615	1.000
RSTAB 8, Definable Stiffness		39.615	1.000