0120 – Single-Mass Oscillation with Dashpot

Description

A single-mass system with dashpot is subjected to a constant loading force F according to **Figure 1**. Determine the deflection u_x and the velocity $\dot{u}_x$ of the dashpot endpoint in test time $t = 3.5$ s. The problem is described by the following parameters.

System Properties	Dashpot	Stiffness	k	2000.000	N/m
		Length	L	0.200	m
		Damping Parameter	c	100.000	Ns/m
	Mass	Weight	m	100.000	kg
Load		Force	F	200.000	N

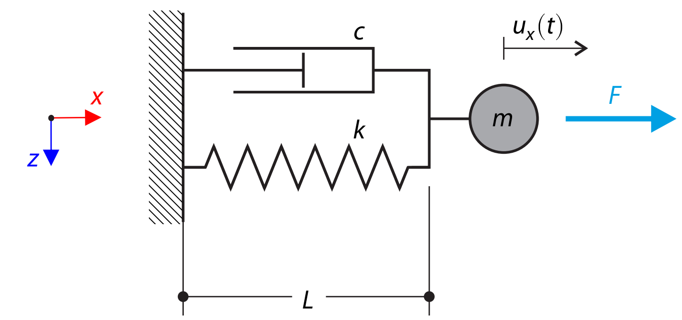

Figure 1: Problem sketch

Analytical Solution

A damped single-mass system is described by the second-order differential equation[1]

$$m\ddot{u}_x + c\dot{u}_x + ku_x = F \,. \qquad (120-1)$$

Using the damping ratio $c_r = \frac{c}{2m\Omega}$ and the natural angular frequency $\Omega = \sqrt{\frac{k}{m}}$, **(120 – 1)** can be rewritten as

$$\ddot{u}_x + 2c_r\Omega\dot{u}_x + \Omega^2 u_x = \frac{F}{m} \,. \qquad (120-2)$$

The homogeneous solution $u_{x,\text{hom}}(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$ of **(120 – 2)** is then found through the characteristic equation

[1] $\dot{u}_x$ and $\ddot{u}_x$ denote the first and second time derivative of u_x, respectively.

$$\lambda^2 + 2c_r\Omega\lambda + \Omega^2 = 0, \tag{120 – 3}$$

where the corresponding roots are

$$\lambda_{1,2} = -c_r\Omega \pm \Omega\sqrt{c_r^2 - 1}. \tag{120 – 4}$$

The desired solution is composed of the homogeneous and particular solution

$$u_x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + \frac{F}{k}, \tag{120 – 5}$$

where the constants C_1 and C_2 are determined from the initial conditions

$$u_x(0) = 0, \tag{120 – 6}$$

$$\dot{u}_x(0) = 0. \tag{120 – 7}$$

The resulting solution of the endpoint deflection u_x reads then

$$u_x(t) = \frac{F}{2k}\left(1 - \frac{c_r}{\sqrt{c_r^2 - 1}}\right) e^{\left(\sqrt{c_r^2-1}-c_r\right)\Omega t} - \frac{F}{2k}\left(3 - \frac{c_r}{\sqrt{c_r^2 - 1}}\right) e^{\left(-\sqrt{c_r^2-1}-c_r\right)\Omega t} + \frac{F}{k}. \tag{120 – 8}$$

The deflection u_x and the velocity $\dot{u}_x$ of the dashpot endpoint at test time $t = 3.5$ s equals to

$$u_x(3.5) \approx 116.874 \text{ mm}, \tag{120 – 9}$$

$$\dot{u}_x(3.5) \approx 11.967 \text{ mm/s}. \tag{120 – 10}$$

RFEM 5 and RSTAB 8 Settings

- Modeled in RFEM 5.14.01 and RSTAB 8.14.01
- The global element size is $l_{FE} = 0.2$ m
- Dashpot member entity (**Figure 2**) is used

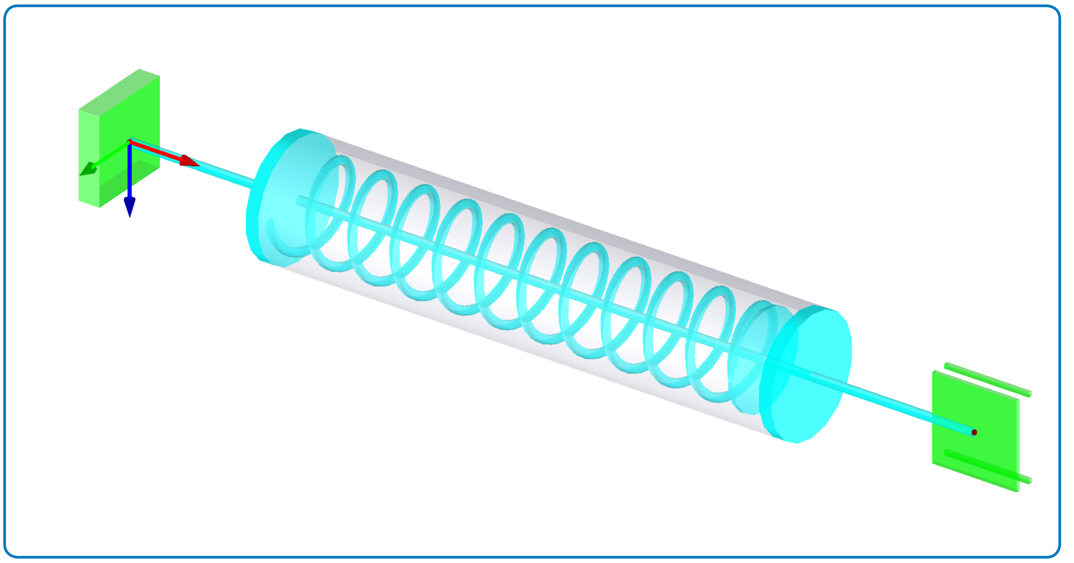

Figure 2: Dashpot representation in RFEM 5 / RSTAB 8

Results

Structure Files	Program	Method
0120.01	RFEM 5 – RF-DYNAM Pro	Linear Implicit Newmark Analysis
0120.02	RFEM 5 – RF-DYNAM Pro	Nonlinear Implicit Newmark Analysis
0120.03	RFEM 5 – RF-DYNAM Pro	Explicit Analysis
0120.04	RSTAB 8 – DYNAM Pro	Linear Implicit Newmark Analysis
0120.05	RSTAB 8 – DYNAM Pro	Explicit Analysis

Model	Analytical Solution	RFEM 5 / RSTAB 8	
	$u_x(3.5)$ [mm]	$u_x(3.5)$ [mm]	Ratio [-]
RFEM 5, Linear Implicit Newmark Analysis	116.874	116.807	0.999
RFEM 5, Nonlinear Implicit Newmark Analysis		116.807	0.999
RFEM 5, Explicit Analysis		116.297	0.995
RSTAB 8, Linear Implicit Newmark Analysis		116.877	1.000
RSTAB 8, Explicit Analysis		116.874	1.000

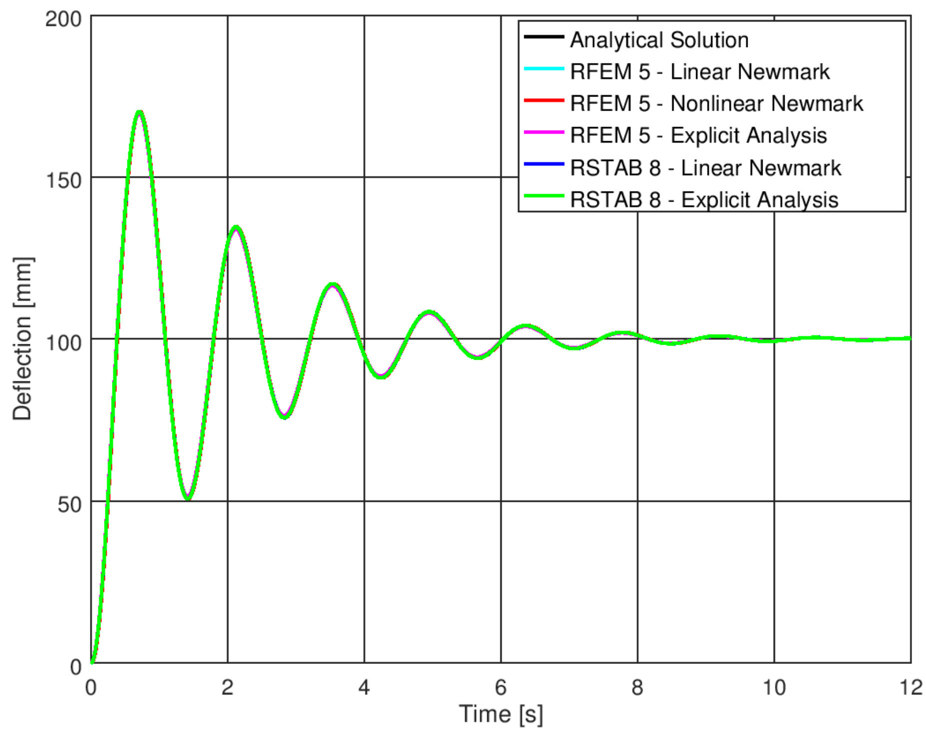

Figure 3: Analytical and RFEM 5 / RSTAB 8 solution - deflection $u_x(t)$

Model	Analytical Solution	RFEM 5 / RSTAB 8	
	$\dot{u}_x(3.5)$ [mm/s]	$\dot{u}_x(3.5)$ [mm/s]	Ratio [-]
RFEM 5, Linear Implicit Newmark Analysis	11.967	13.919	1.163
RFEM 5, Nonlinear Implicit Newmark Analysis		13.918	1.163
RFEM 5, Explicit Analysis		11.252	0.940
RSTAB 8, Linear Implicit Newmark Analysis		12.168	1.016
RSTAB 8, Explicit Analysis		11.967	1.000

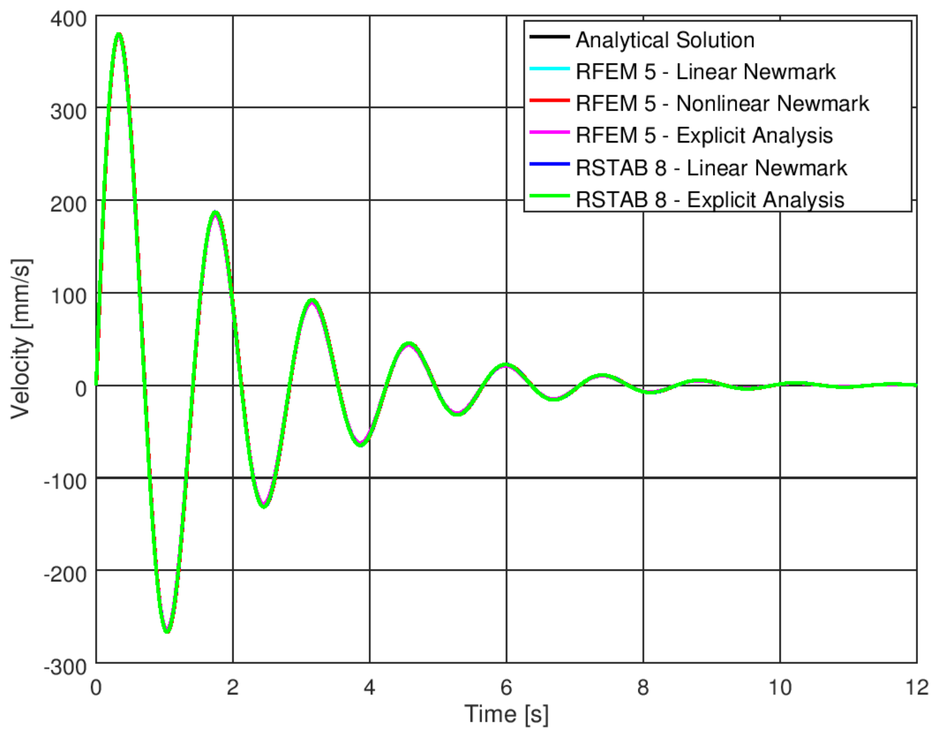

Figure 4: Analytical and RFEM 5 / RSTAB 8 solution - velocity $\dot{u}_x(t)$