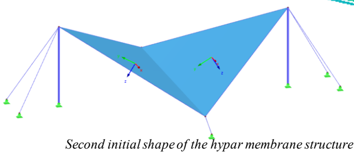
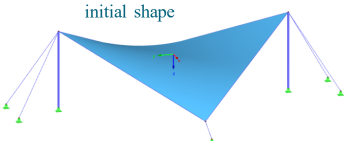
FORM-FINDING

Essential nature of the form-finding

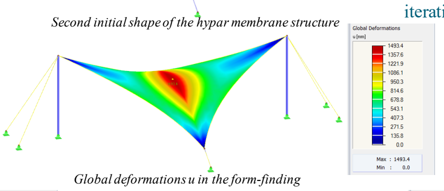
- The shape is given by the equilibrium of forces and boundary conditions
- The shape is independent of the material
- The resulting shape is independent of the initial shape



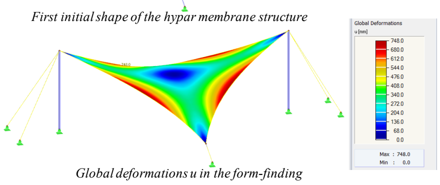
Second initial shape of the hypar membrane structure



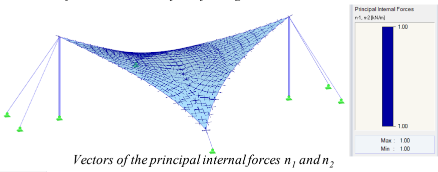
First initial shape of the hypar membrane structure



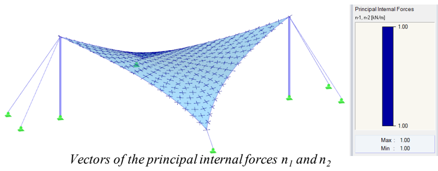
Global deformations u in the form-finding



Global deformations u in the form-finding



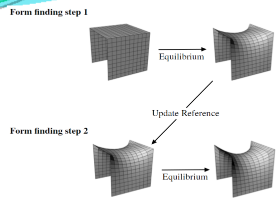
Vectors of the principal internal forces n_1 and n_2



Vectors of the principal internal forces n_1 and n_2

Nonlinear calculation

- The shape corresponding to the specific equilibrium of forces is searched in the iterative process



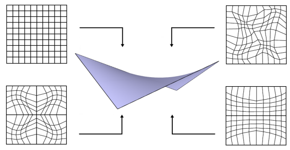
Iterations of the form-finding process

The equation of shape in equilibrium

- $\frac{\partial \pi}{\partial d} = \frac{\partial \pi^{int}}{\partial d} + \frac{\partial \pi^{ext}}{\partial d} = \int_{\Omega_0} S : \delta E \, d\Omega_0 - \int_{\Omega_0} q \cdot \delta d \, d\Omega_0 = \int_{\Omega} \sigma : \delta e \, d\Omega - \int_{\Omega} q \cdot \delta d \, d\Omega = 0$

Special phenomena of the form-finding

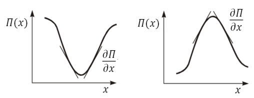
- The tangential redistribution of the nodes does not change the shape, thus the balance. There is no unique solution.



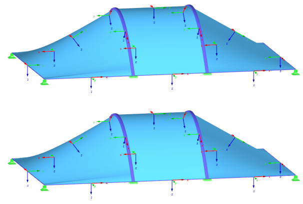
Arbitrarily deformed meshes for the same surface geometry

Form-finding of tensioned and compressed structures

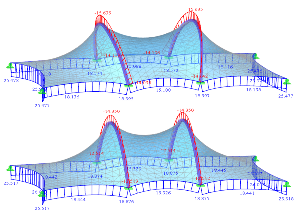
- The form-finding of tensioned structures lead to the finding of stable equilibrium position
- The form-finding of compressed structures lead to the finding of unstable equilibrium position
- Structures with both requirement can be subjected to the form-finding and the position of arches can be optimized to carry compression only



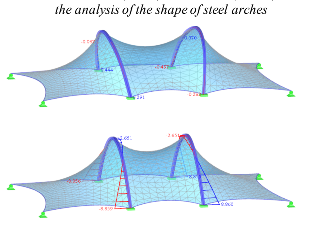
Stable and unstable equilibrium position



Initial shape of the membrane structures; the structure with (above) and without (bellow) the analysis of the shape of steel arches



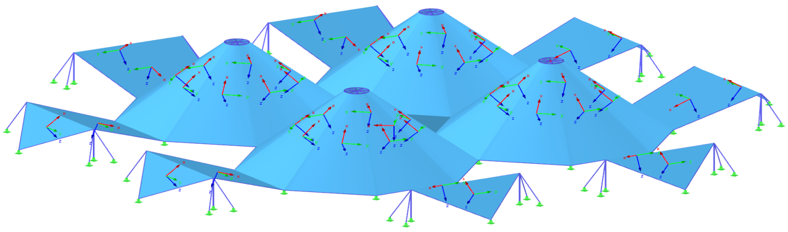
Normal forces N; the structure with (above) and without (bellow) the analysis of the shape of steel arches



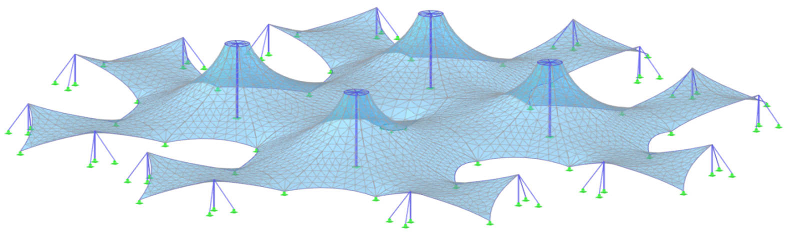
Bending moments M_z; the structure with (above) and without (bellow) the analysis of the shape of steel arches

Form-finding with consideration of supporting structure

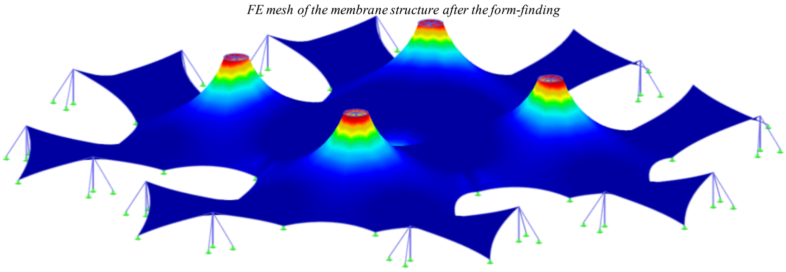
- The supporting structure can be calculated in the same time as the form-finding
- Defined load can be taken into account as for example the self weight



Initial shape of the membrane structure with the warp/weft orientation displaying



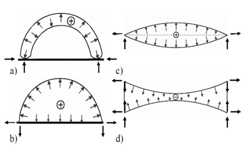
FE mesh of the membrane structure after the form-finding



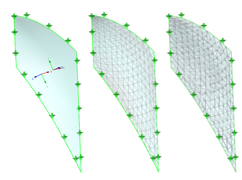
Principal internal forces n_1

Pneumatic structures

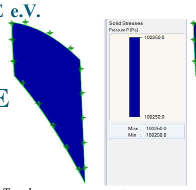
- The gas can be part of the form-finding process
- The required pressure can be the task as well as the required volume



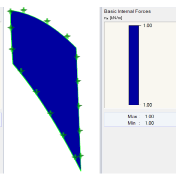
Pneumatic prestressed and stabilized structures (+ overpressure, - low pressure)



ETFE cushion (left), FE mesh of the layers (middle), FE mesh of the air chamber (right)



Total pressure $p = p_a + p_o$ (p_a...atmospheric pressure + p_o...overpressure)



Basic internal forces n_x

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AUTHOR Ing. ROSTISLAV LANG
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SEPTEMBER 2017
DESSAU-ROSSLAU
GERMANY

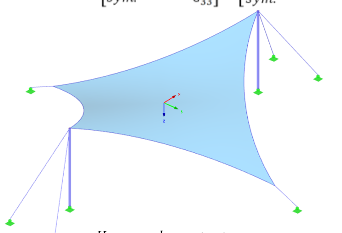
STRUCTURAL ANALYSIS

Nonlinear analysis of hypar structure

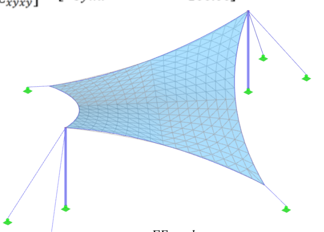
- Loads and load combinations as described
- Orthotropic linear elastic material model with values

$$E_x = 1000.0\frac{kN}{m}, E_y = 800.0\frac{kN}{m}, G_{xy} = 100.0\frac{kN}{m}, \nu_{xy} = 0.10, \nu_{yx} = 0.08$$

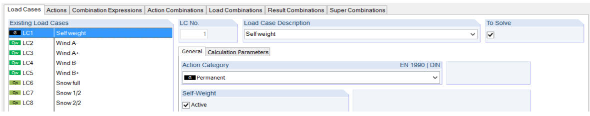
$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ & C_{22} & C_{23} \\ sym. & & C_{33} \end{bmatrix} = \begin{bmatrix} C_{xxxx} & C_{xxyy} & C_{xxxy} \\ & C_{yyyy} & C_{yyxy} \\ sym. & & C_{xyxy} \end{bmatrix} = \begin{bmatrix} 1008.06 & 80.65 & 0.00 \\ & 806.45 & 0.00 \\ sym. & & 100.00 \end{bmatrix} \frac{kN}{m}$$



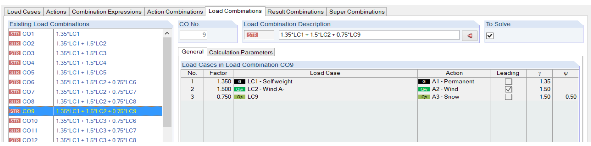
Hypar membrane structure



FE mesh



Definition of load cases

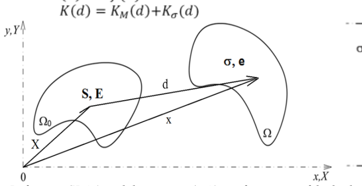


Load combinations (ULS and SLS)

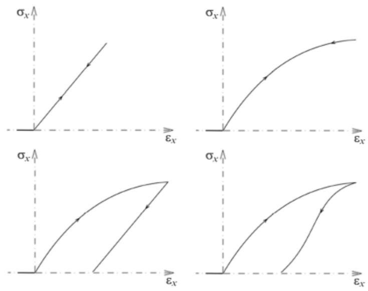
Nonlinear analysis

- Geometrical nonlinearity
- Material nonlinearity

$$K(d)d = f(d)$$
$$K(d) = K_M(d) + K_\sigma(d)$$



Reference (X,Ω_0) and the current (x,Ω) configuration of the body



Elastic, non-linear elastic, elastic-plastic and general material models

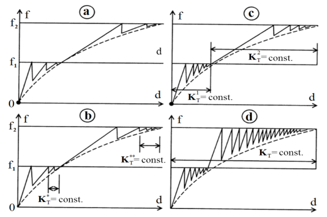
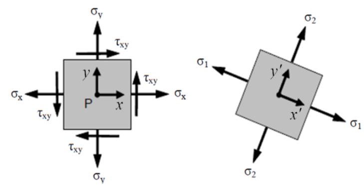
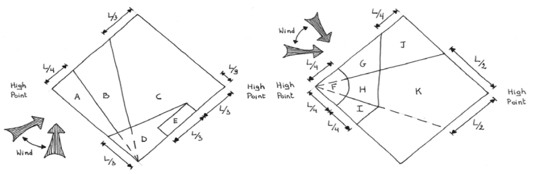


Diagram of the Newton-Raphson iterative method a) and its three modifications b),c),d)

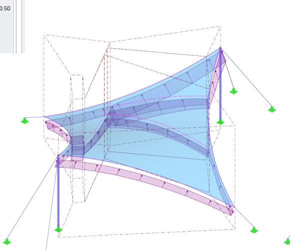


Stress state: stresses in the planar axis direction, main stresses in the main directions

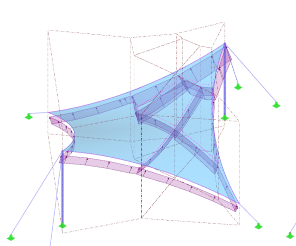
$$\frac{\partial\pi}{\partial d} = \frac{\partial\pi^{int}}{\partial d} + \frac{\partial\pi^{ext}}{\partial d} = \int_{\Omega_0} S : \delta E \, d\Omega_0 - \int_{\Omega_0} q \cdot \delta d \, d\Omega_0 = \int_{\Omega} \sigma : \delta e \, d\Omega - \int_{\Omega} q \cdot \delta d \, d\Omega = 0$$



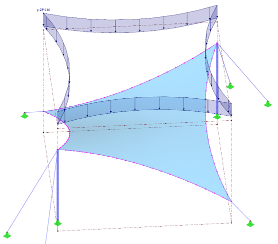
C_p zones definition on the hypar structure



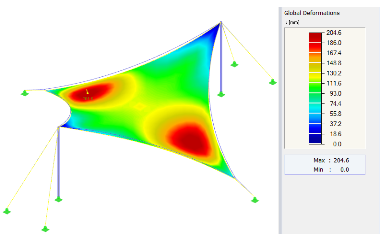
Load case LC2 (Wind A-)



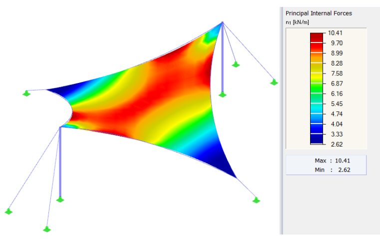
Load case LC4 (Wind B-)



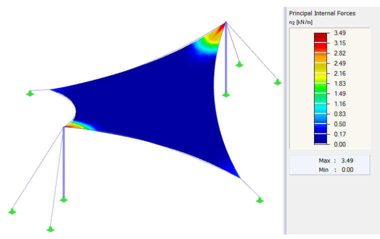
Load case LC6 (Snow full)



Global deformations u (CO18)



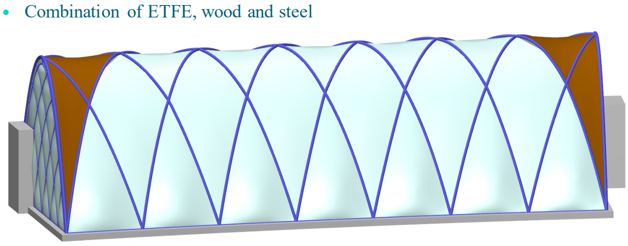
Main internal forces n_1 (CO18=1.35LC1+1.50LC6)



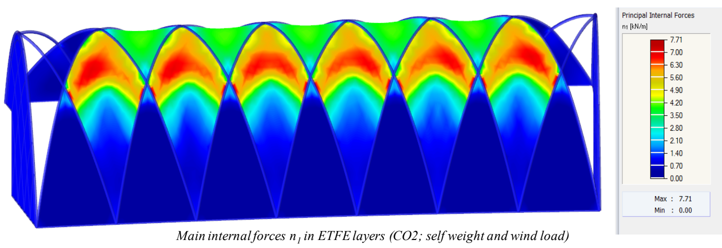
Main internal forces n_2 (CO18=1.35LC1+1.50LC6)

Nonlinear analysis of greenhouse structure

- Combination of ETFE, wood and steel



Geometry of the greenhouse structure



Main internal forces n_1 in ETFE layers (CO2; self weight and wind load)

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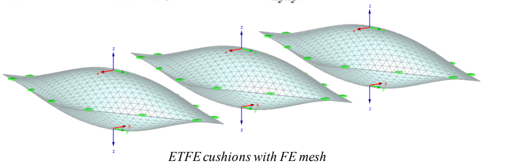
GENERATION OF CUTTING PATTERNS

Flattening of the greenhouse cushion

- Physical Squashing with Energy Minimization
- Influence of the pattern size on the distortions caused by flattening
- Material considered in the flattening process

$E = 900.0\,MPa, G = 310.0\,MPa, \nu = 0.452$

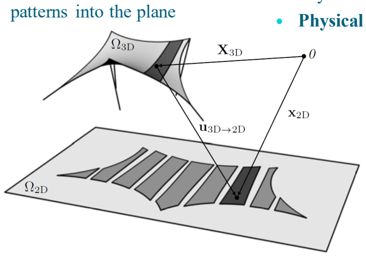
$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ & C_{22} & C_{23} \\ sym. & & C_{33} \end{bmatrix} = \begin{bmatrix} C_{xxxx} & C_{xxyy} & C_{xxxy} \\ & C_{yyyy} & C_{yyxy} \\ sym. & & C_{xyxy} \end{bmatrix} = \begin{bmatrix} 339.18 & 153.18 & 0.00 \\ & 339.18 & 0.00 \\ sym & & 93.00 \end{bmatrix} \frac{kN}{m}$$



ETFE cushions with FE mesh

Generation of cutting patterns

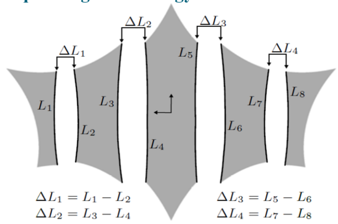
- Dividing the surface into spatial patterns by cutting lines
- Flattening of the spatial patterns into the plane



The flattening process

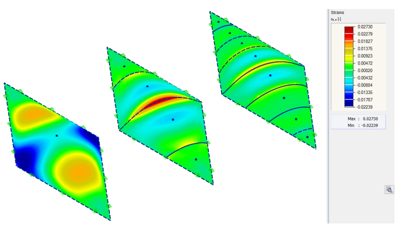
Some of flattening methods

- Simple Triangulation Method
- Mathematical Squashing by Least Square Approach
- Physical Squashing by Least Square Approach
- **Physical Squashing with Energy Minimization**

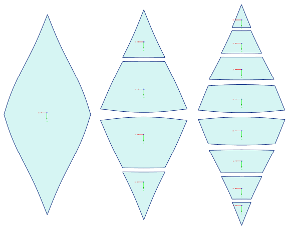


Ensuring the same lengths of the boundary lines of the adjacent patterns

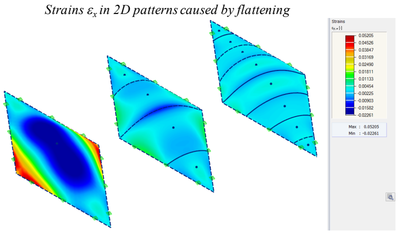
$$\frac{\partial \Pi}{\partial d} = \frac{\partial \Pi^{int}}{\partial d} = \frac{\partial(\Pi^{int}_{3D\to 2D} + \Pi^{int}_{pre})}{\partial d} = \int_{\Omega_{3D}} (S_{3D\to 2D} + S_{pre}) : \delta E_{3D\to 2D}\, d\Omega_{3D} = \int_{\Omega_{2D}} (\sigma_{3D\to 2D} + \sigma_{pre}) : \delta e_{3D\to 2D}\, d\Omega_{2D} = 0$$



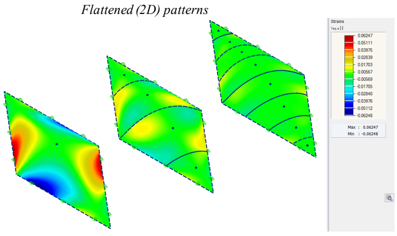
Strains ε_x in 2D patterns caused by flattening



Flattened (2D) patterns



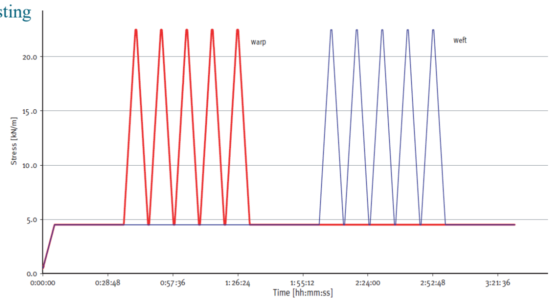
Strains ε_y in 2D patterns caused by flattening



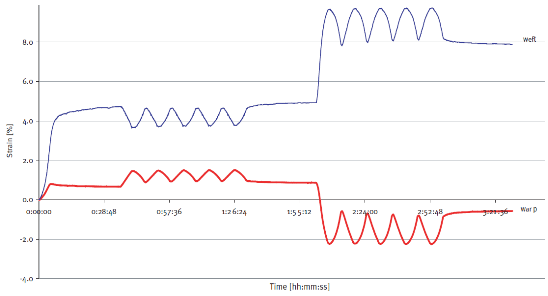
Strains ε_{xy} in 2D patterns caused by flattening (displayed on spatial (3D) patterns for having compact model of all patterns)

Compensation

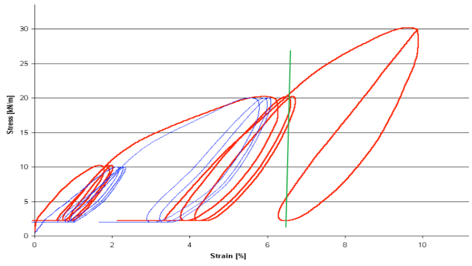
- Significant plastic deformation in first loop of fabric loading
- Biaxial testing



Biaxial test: load history



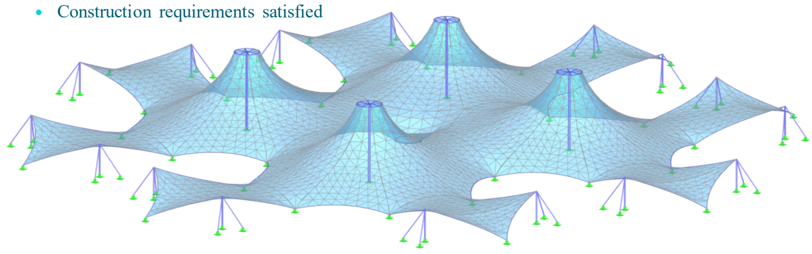
Biaxial test: measured strains



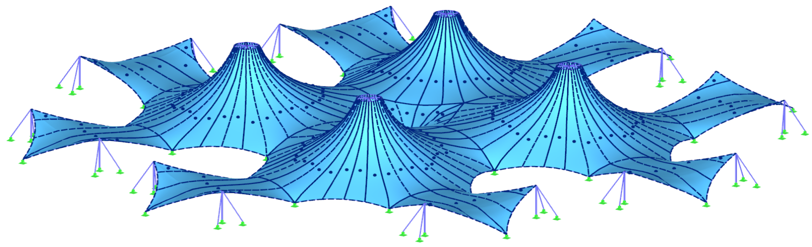
Biaxial test: stress-strains diagram

Generation of cutting patterns

- Construction requirements satisfied



FE mesh of the membrane structure



Spatial patterns

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