Verification Example

Program: RFEM 5, RF-STAGES, RF-HISTORY

Category: Isotropic Linear Elasticity, Large Deformation Analysis, Solid

Verification Example: 0041 – Uni-Axially Stretched Beam

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Description

A vertical cantilever with a square cross-section is loaded at the top by the tensile pressure p. The other side is fully fixed; see figure **Figure 1** for further details. The cantilever consists of an isotropic Hookean material. Determine the extremal deflection u_X, u_Y and u_Z in global X, Y and Z-axis considering the Large deformation analysis. Determine also the proper reaction forces. The aim of this example is to compare the exact analytical solution with RFEM 5 solution, RF-STAGES solution and RF-LOAD HISTORY solution respectively. The problem is described by the following set of input parameters.

Material	Rubber	Modulus of Elasticity	E	75.000	MPa
		Poisson's Ratio	ν	0.499	–
		Shear Modulus	G	25.000	MPa
Geometry	Cantilever	Length	L	1.000	m
		Height	h	0.005	m
Load		Pressure	p	−5.000	MPa

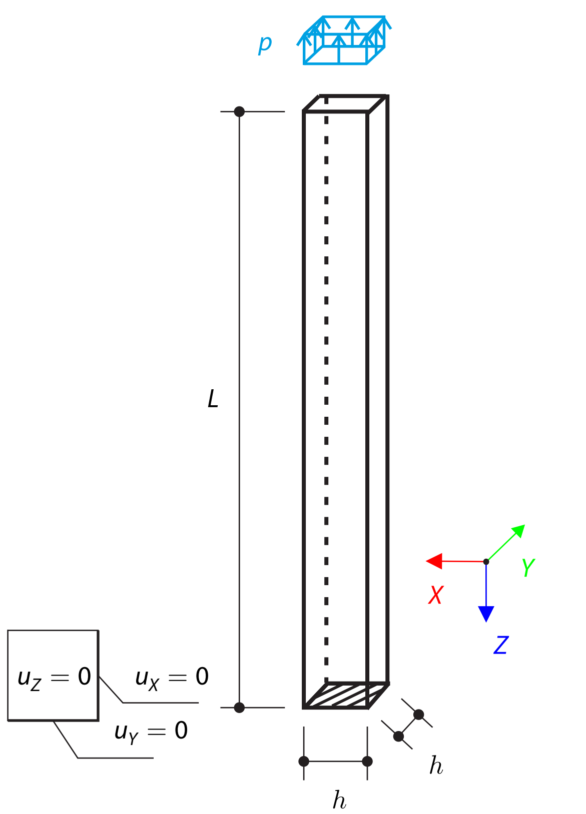

Figure 1: Problem sketch

Analytical Solution

Let us derive formula for cantilever elongation. It is convenient to write the Hooke law in the following form considering infinitesimal elongation dz.

$$\mathrm{d}\sigma(z) = E\frac{\mathrm{d}z}{-L+z} \tag{41 – 1}$$

Integration from 0 to Δz over z yields

$$\sigma(\Delta z) = \int_0^{\Delta z} E\frac{\mathrm{d}z}{-L+z} = E(\ln|-L+\Delta z| - \ln|L|) \tag{41 – 2}$$

which results in

$$\Delta z = L\left(1 - e^{\frac{\sigma(\Delta z)}{E}}\right) = \left(1 - e^{\frac{-p}{E}}\right) \tag{41 – 3}$$

Evaluating formula **(41 – 3)** leads to

$$\Delta z = -68.939 \text{ mm} \tag{41 – 4}$$

Recalling the general formula for Poisson's ratio

$$\nu = -\frac{\mathrm{d}\epsilon_x}{\mathrm{d}\epsilon_z} \tag{41 – 5}$$

can be transformed to the following differential equality

$$\frac{\mathrm{d}x}{x} = -\nu\frac{\mathrm{d}z}{z} \tag{41 – 6}$$

which can be solved by formal integration

$$\int_h^{h+\Delta x} \frac{\mathrm{d}x}{x} = -\nu \int_{-L}^{-L+\Delta z} \frac{\mathrm{d}z}{z} \tag{41 – 7}$$

Simple algebraic manipulation leads to

$$\Delta x = -h\left[1 - \left(1 - \frac{\Delta z}{L}\right)^{-\nu}\right] \tag{41 – 8}$$

Evaluating formula **(41 – 7)** produces

$$\Delta x = -0.164 \text{ mm} \tag{41 – 9}$$

If we take the symmetry of the problem into the account, we get

$$\Delta y = \Delta x = -0.164 \text{ mm} \tag{41 – 10}$$

The surface area of both cantilever ends has changed accordingly to

$$A = (h + \Delta x)^2 \tag{41 – 11}$$

Therefore, the magnitude of the vertical reaction force acting on the bottom surface has following value

$$R = |p|A = |p|(h + \Delta x)^2 = 0.117 \text{ kN} \tag{41 – 12}$$

RFEM 5 Settings

- Modeled in RFEM 5.16.01
- The element size is $l_{\text{FE}} = 0.1$ m
- The element type is solid
- Large deformation analysis is considered
- Isotropic linear elastic material model is used

Results

Structure File	Program	Description
0041.01	RFEM 5	100 increments
0041.02	RF-STAGES	2 Stages , 50 increments per stage
0041.03	RF-LOAD HISTORY	2 Steps , 50 increments per step

Analytical Solution	RFEM 5		RF- STAGES		RF- LOAD HISTORY	
u_Z [mm]	u_Z [mm]	Ratio [-]	u_Z [mm]	Ratio [-]	u_Z [mm]	Ratio [-]
−68.939	−68.892	0.999	−68.892	0.999	−68.892	0.999

Analytical Solution	RFEM 5		RF- STAGES		RF- LOAD HISTORY	
u_X [mm]	u_X [mm]	Ratio [-]	u_X [mm]	Ratio [-]	u_X [mm]	Ratio [-]
−0.164	−0.164	1.000	−0.164	1.012	−0.164	1.000

Analytical Solution	RFEM 5		RF- STAGES		RF- LOAD HISTORY	
R [kN]	R [kN]	Ratio [-]	R [kN]	Ratio [-]	R [kN]	Ratio [-]
0.117	0.117	1.000	0.117	1.000	0.117	1.000