Program: RFEM 5, RSTAB 8

Category: Large Deformation Analysis, Isotropic Linear Elasticity, Member

Verification Example: 0044 – Geometric Non-Linearity of Member with Spring

0044 – Geometric Non-Linearity of Member with Spring

Description

Slightly sloped member is loaded with the force *F* and held by spring with the stiffness *k* at one end and supported according to the **Figure 1**. Assuming large deformations and neglecting member's self-weight, determine its maximum upward deflection u_z.

Material	Steel	Modulus of Elasticity	E	210.000	GPa
		Poisson's Ratio	ν	0.300	–
Geometry	Beam	Height	h	0.100	m
		Width	b	0.100	m
		Horizontal Length	L_x	2.500	m
		Vertical Length	L_z	0.025	m
Spring		Stiffness	k	1	kN/m
Load		Force	F	1	kN

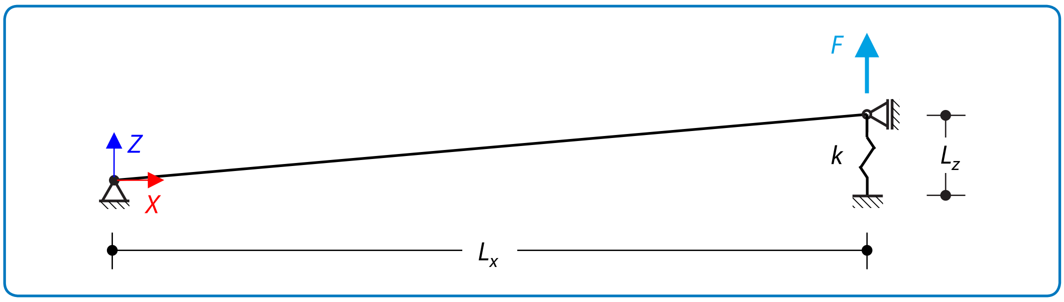

Figure 1: Problem sketch [1]

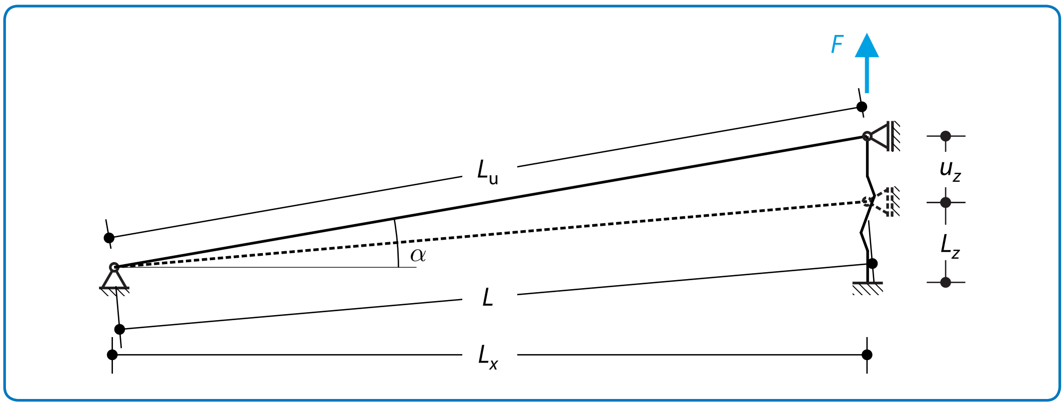

Figure 2: Deformed member

Analytical Solution

According to the **Figure 2** vertical equilibrium condition can be compiled:

$$F = N \sin \alpha + k u_z = \frac{N(L_z + u_z)}{L_u} + k u_z \approx \frac{N(L_z + u_z)}{L} + k u_z \qquad (44-1)$$

where N is the normal force of the member, which can be expressed as:

$$N = EA\varepsilon \qquad (44-2)$$

where $A = bh$ is the cross-section area and ε is the axial strain of the member. Assuming that α is small, the strain ε can be evaluated by Pythagoras's theorem as follows:

$$\varepsilon = \frac{\sqrt{(L_z + u_z)^2 + L_x^2} - \sqrt{L_x^2 + L_z^2}}{\sqrt{L_x^2 + L_z^2}} \approx \frac{1}{L_x^2}\left(L_z u_z + \frac{1}{2} u_z\right) \approx \frac{1}{L^2}\left(L_z u_z + \frac{1}{2} u_z\right) \qquad (44-3)$$

Using formulae **(44 – 2)** and **(44 – 3)**, the cubic function of the displacement u_z can be obtained from equation **(44 – 1)**:

$$u_z^3 + 3L_z u_z^2 + 2\left(L_z^2 + \frac{kL^3}{EA}\right) u_z - \frac{2FL^3}{EA} = 0 \qquad (44-4)$$

The equation **(44 – 4)** has only one real root:

$$u_z = 7.792 \text{ mm} \qquad (44-5)$$

RFEM 5 and RSTAB 8 Settings

- Modeled in version RFEM 5.04.0058 and RSTAB 8.04.0058
- The element size is $l_{FE} = 0.250$ m
- Large deformation analysis is considered
- The number of increments is 1
- Isotropic linear elastic material model is used
- Shear stiffness of members is activated
- Member division for large deformation or post-critical analysis is activated

Results

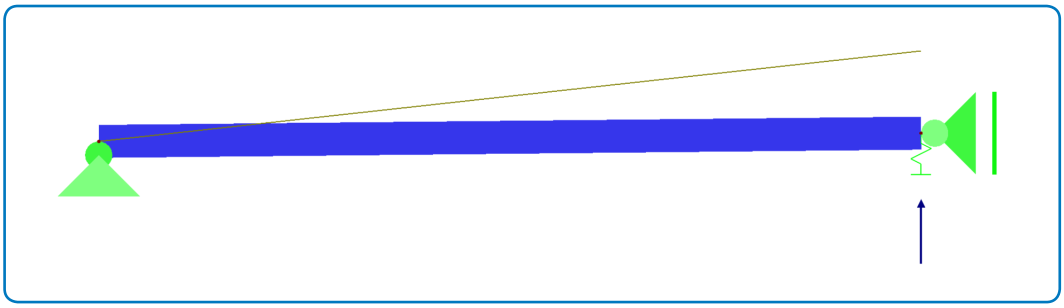

Figure 3: RFEM 5 Model

Structure File	Program
0044.01	RFEM 5
0044.02	RSTAB 8

As can be seen from the following comparison, an excellent agreement of analytical solution with numerical outputs was achieved:

Analytical Solution	RFEM 5		RSTAB 8	
u_z [mm]	u_z [mm]	Ratio [-]	u_z [mm]	Ratio [-]
7.792	7.792	1.000	7.792	1.000

References

[1] CRISFIELD, M. A. *Non-linear Finite Element Analysis of Solids and Structures*. John Wiley & Sons Ltd..