Program: RFEM 5, RSTAB 8

Category: Geometrically Linear Analysis, Large Deformation Analysis, Isotropic Linear Elasticity, Member

Verification Example: 0052 – Cantilever with the Moment Loading at the Free End

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Description

A cantilever is loaded by the moment M at its free end. Using the geometrically linear analysis and the large deformation analysis and neglecting beam's self-weight, determine the maximum deflections u_x and u_z at the free end.

Material	Steel	Modulus of Elasticity	E	210.000	GPa
		Shear Modulus	G	81.000	GPa
Geometry	Beam	Length	L	4000.000	mm
		Diameter	d	42.400	mm
		Wall Thickness	t	4.000	mm
Load		Bending Moment	M	3.400	kNm

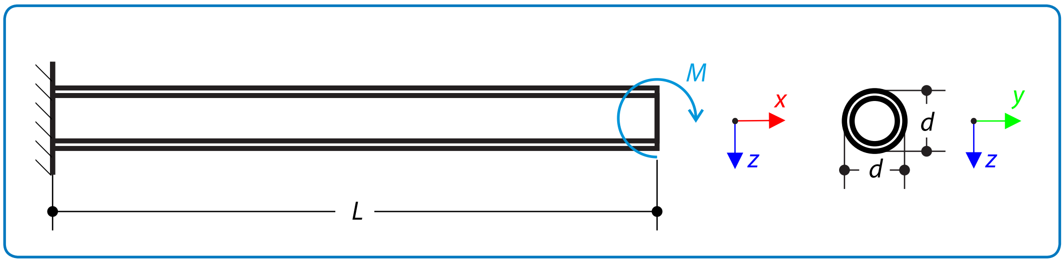

Figure 1: Problem sketch [1]

Analytical Solution

Geometrically Linear Analysis

Considering the geometrically linear analysis, the problem can be solved according to the Euler-Bernoulli equation

$$u_z''(x) = -\frac{M}{EI_y} \qquad (52-1)$$

with boundary conditions

$$u_z(0) = u_z'(0) = 0 \qquad (52-2)$$

where I_y is the second moment of inertia around y axis (see **Figure 1**):

$$I_y = \frac{\pi \left[d^4 - (d-2t)^4\right]}{64} \approx 89908.5\,\text{mm}^4 \qquad (52-3)$$

The equation **(52 – 1)** has the following solution:

$$u_{z,\max} = \frac{ML^2}{2EI_y} \approx 1.441\,\text{m} \qquad (52-4)$$

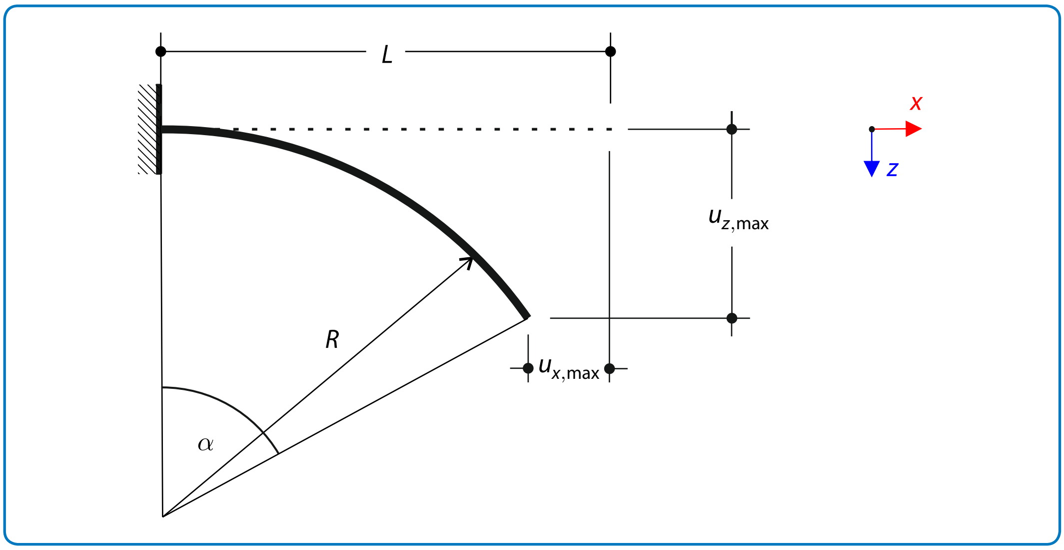

Figure 2: The large deformation theory

Large Deformational Analysis

A beam in the large deformation analysis is described by the nonlinear differential equation

$$\kappa(x) = \frac{u_z''(x)}{\left[1 + (u_z'(x))^2\right]^{\frac{3}{2}}} = -\frac{M}{EI_y} \qquad (52-5)$$

which is difficult to solve in general. However, the term on the right-hand side is constant and consequently the left-hand side, which is nothing else then the beam curvature κ, is also constant. The only curve which has constant curvature is a circle, therefore, the solution to this problem is a circle arc of radius R. We get

$$u_{x,\max} = R\sin\alpha - L \qquad (52-6)$$

$$u_{z,\max} = R(1-\cos\alpha) \qquad (52-7)$$

where

$$R = \left|\frac{1}{\kappa(x)}\right| = \frac{EI_y}{M} \approx 5.553\,\text{m} \qquad (52-8)$$

is the radius of the circular arc. The angle of the circular arc α equals to $\alpha = \frac{L}{R} \approx 0.72$ rad.

RFEM 5 and RSTAB 8 Settings

- Modeled in version RFEM 5.03.0050 and RSTAB 8.03.0050
- The element size is $l_{FE} = 0.400$ m
- The number of increments is 1
- Isotropic linear elastic material model is used
- Shear stiffness of members is activated
- Member division for large deformation or post-critical analysis is activated

Results

Structure File	Program	Method of Analysis
0052.01	RFEM 5	Geometrically Linear Analysis
0052.02	RFEM 5	Large Deformation Analysis
0052.03	RSTAB 8	Geometrically Linear Analysis
0052.04	RSTAB 8	Large Deformation Analysis

An excellent agreement of the analytical results with the numerical outputs were achieved:

Method of Analysis	Analytical Solution	RFEM 5		RSTAB 8	
	$u_{x,\max}$ [m]	$u_{x,\max}$ [m]	Ratio [-]	$u_{x,\max}$ [m]	Ratio [-]
Geometrically Linear	0.000	0.000	-	0.000	-
Large Deformation	-0.337	-0.338	1.003	-0.337	1.000

Method of Analysis	Analytical Solution	RFEM 5		RSTAB 8	
	$u_{z,\max}$ [m]	$u_{z,\max}$ [m]	Ratio [-]	$u_{z,\max}$ [m]	Ratio [-]
Geometrically Linear	1.441	1.441	1.000	1.441	1.000
Large Deformation	1.379	1.380	1.001	1.380	1.001

References

[1] LUMPE, G. and GENSICHEN, V. *Evaluierung der linearen und nichtlinearen Stabstatik in Theorie und Software: Prüfbeispiele, Fehlerursachen, genaue Theorie*. Ernst.