Program: RFEM 5, RFEM 6

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Plate

Verification Example: 0065 – Two–Layered Thick–Walled Vessel

0065 – Two–Layered Thick–Walled Vessel

Description

A two–layered thick–walled vessel is loaded by inner and outer pressure. The vessel is open, thus there is no axial stress. The problem is modeled as a quarter model, see **Figure 1**, and is described by the following set of parameters.

Material	Inner vessel	Modulus of Elasticity	E	1.000	MPa
		Poisson's Ratio	ν	0.250	–
	Outer vessel	Modulus of Elasticity	E	0.500	MPa
		Poisson's Ratio	ν	0.250	–
Geometry		Inner radius	r_1	200.000	mm
		Middle radius	r_m	250.000	mm
		Outer radius	r_2	300.000	mm
Load		Inner pressure	p_1	60.000	kPa
		Outer pressure	p_2	10.000	kPa

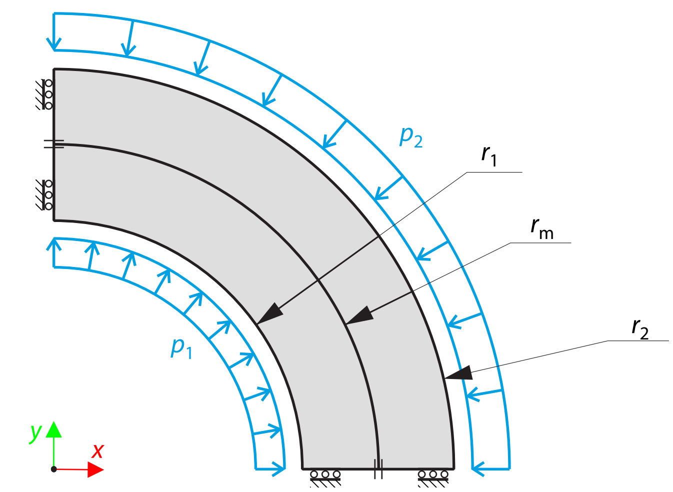

Figure 1: Problem Sketch

Determine the radial deflection of the inner and outer radii $u_r(r_1)$, $u_r(r_2)$ and the pressure (radial stress) in the middle radius p_m. Self-weight is neglected.

Analytical Solution

The analytical solution of the given problem is analogous to the analytical solution of Verification Example 0064 – Thick–Walled Vessel [1]. The general radial deflection of the vessel is given by

$$u_r(r) = \frac{r}{E}\left[\sigma_t(r) - \nu\sigma_r(r)\right] \tag{65 – 1}$$

which defines also the radial deflection of the middle radius of both the inner and outer vessel, namely

$$u_r(r_m) = \frac{r_m}{E_1}\left[K_1 + \frac{C_1}{r_m^2} - \nu\left(K_1 - \frac{C_1}{r_m^2}\right)\right] \tag{65 – 2}$$

$$u_r(r_m) = \frac{r_m}{E_2}\left[K_2 + \frac{C_2}{r_m^2} - \nu\left(K_2 - \frac{C_2}{r_m^2}\right)\right] \tag{65 – 3}$$

Constants K_1, C_1, K_2 and C_2 are calculated subsequently for each vessel from the corresponding radii and boundary pressures, for more details see [1]. Using these equations, the pressure in the interface p_m can be determined.

$$p_m = \frac{2(E_1 p_2 r_2^2 (r_1^2 - r_m^2) + E_2 p_1 r_1^2 (r_m^2 - r_2^2))}{E_2(r_2^2 - r_m^2)\left[(1+\nu)r_1^2 + (1-\nu)r_m^2\right] + E_1(r_m^2 - r_1^2)\left[(1+\nu)r_2^2 + (1-\nu)r_m^2\right]} = 21.655 \text{ kPa} \tag{65 – 4}$$

In turn, the radial displacements $u_r(r_1)$, $u_r(r_2)$ can be calculated with the help of **(65 – 4)**,

$$u_r(r_1) = \frac{r_1}{E_1}\left[K_1 + \frac{C_1}{r_1^2} - \nu\left(K_1 - \frac{C_1}{r_1^2}\right)\right] = 33.605 \text{ mm} \tag{65 – 5}$$

$$u_r(r_2) = \frac{r_2}{E_2}\left[K_2 + \frac{C_2}{r_2^2} - \nu\left(K_2 - \frac{C_2}{r_2^2}\right)\right] = 27.287 \text{ mm} \tag{65 – 6}$$

see **Figure 2** for the former one.

RFEM Settings

- Modeled in RFEM 5.06 and RFEM 6.01
- The element size is $l_{FE} = 0.002$ m
- Isotropic linear elastic material model is used

Results

Structure Files	Program
0065.01	RFEM 5, RFEM 6

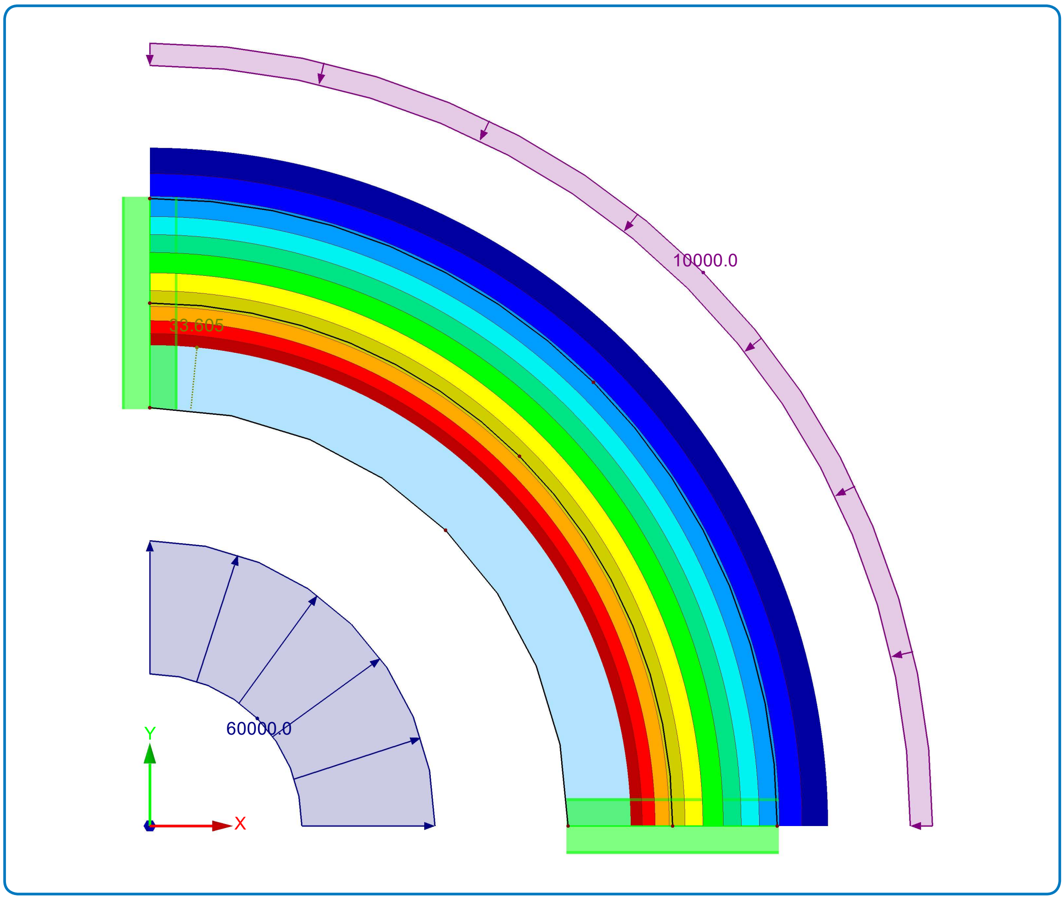

Figure 2: Results in RFEM - deflection $u_r(r_1)$

Quantity	Analytical Solution	RFEM 5	Ratio	RFEM 6	Ratio
p_m [kPa]	21.655	21.648	1.000	21.663	1.000
$u_r(r_1)$ [mm]	33.605	33.605	1.000	33.602	1.000
$u_r(r_2)$ [mm]	27.287	27.287	1.000	27.283	1.000

References

[1] DLUBAL SOFTWARE GMBH, *Verification Example 0064 – Thick-Walled Vessel*. 2016.