Program: RFEM 5, RF-DYNAM Pro

Category: Geometrically Linear Analysis, Isotropic Linear Elasticity, Dynamics, Plate, Solid

Verification Example: 0110 – Natural Vibrations of Rectangular Plate

0110 – Natural Vibrations of Rectangular Plate

Description

A rectangular steel plate of dimensions a, b and thickness h is simply supported at its edges according to **Figure 1**. Determine the natural frequencies of the rectangular plate. The problem is described by the following parameters.

Material	Steel	Modulus of Elasticity	E	210000.0	MPa
		Poisson's Ratio	ν	0.300	–
		Density	ρ	7850.000	kgm^{-3}
Geometry		Width	a	1.000	m
		Length	b	1.500	m
		Thickness	h	0.010	m

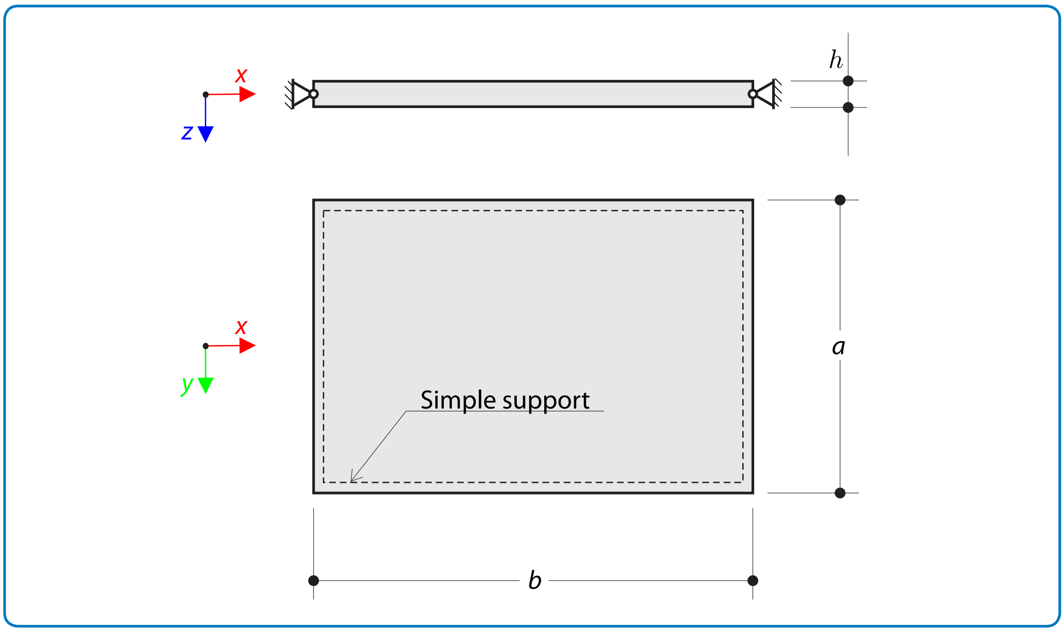

Figure 1: Problem Sketch

Analytical Solution

Free vibrations of a rectangular plate are described by the following differential equation for the z-directional deflection of the plate $u_z(x, y, t)$

$$\frac{\partial^4 u_z}{\partial x^4} + 2\frac{\partial^4 u_z}{\partial x^2 \partial y^2} + \frac{\partial^4 u_z}{\partial y^4} + \frac{1}{c^2}\frac{\partial^2 u_z}{\partial t^2} = 0 \qquad (110 – 1)$$

The speed of the wave propagation c is given by the density of the plate material ρ, plate thickness h and the plate modulus D

$$c = \sqrt{\frac{D}{\rho h}}, \qquad D = \frac{Eh^3}{12(1-\nu^2)} \tag{110 – 2}$$

The solution is sought for using separation of variables

$$u_z(x,y,t) = X(x)Y(y)T(t) \tag{110 – 3}$$

Substitution into **(110 – 1)** yields[1]

$$-\frac{\ddot{T}}{T} = c^2\left(\frac{X^{(4)}}{X} + 2\frac{X''Y''}{XY} + \frac{Y^{(4)}}{Y}\right) = \Omega^2 \tag{110 – 4}$$

The left-hand side depends on time t, while the right-hand side only on the spatial coordinates x and y. Thus both sides have to be equal to a constant Ω^2.

The first part of **(110 – 4)**

$$\ddot{T} + \Omega^2 T = 0 \tag{110 – 5}$$

admits a soltuion in the following form

$$T(t) = A\sin(\Omega t) + B\cos(\Omega t) \tag{110 – 6}$$

where the constants A, B depend on the initial conditions.

The second part of **(110 – 4)** can be rewritten into the biharmonic equation

$$\frac{X^{(4)}}{X} + 2\frac{X''Y''}{XY} + \frac{Y^{(4)}}{Y} = \frac{\Omega^2}{c^2} \tag{110 – 7}$$

According to [1], for a simply supported rectangular plate, solutions satisfying the boundary conditions take the following form

$$X_m(x) = \sin\left(\frac{m\pi x}{a}\right), \qquad m = 1,2,3,\ldots \tag{110 – 8}$$

$$Y_n(y) = \sin\left(\frac{n\pi y}{b}\right), \qquad n = 1,2,3,\ldots \tag{110 – 9}$$

Substituting these functions into **(110 – 7)**, the constants Ω_{mn} are determined

[1] The dashed notation indicates the derivative with respect to appropriate coordinate, e.g. $X'' = \frac{\mathrm{d}^2X(x)}{\mathrm{d}x^2}$. The dotted notation indicates the derivative with respect to time t.

$$\Omega_{mn} = c\pi^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \tag{110 – 10}$$

Considering that $\Omega_{mn} = 2\pi f_{mn}$, the natural frequencies of the rectangular plate can be calculated according to the following equation

$$f_{mn} = \frac{\pi}{2} \sqrt{\frac{D}{\rho h}} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \tag{110 – 11}$$

The first six natural frequencies, for $mn = 11, 12, 21, 13, 22, 23$, are shown in the result table.

RFEM 5 Settings

- Modeled in RFEM 5.07.05
- The element size is $l_{FE} = 0.010$ m for the entity Plate
- The element size is $l_{FE} = 0.050$ m for the entity Solid
- Layered mesh is used for the entity Solid with 4 layers
- Isotropic linear elastic material model is used

Results

Structure Files	Program	Entity
0110.01	RF-DYNAM Pro	Plate
0110.02	RF-DYNAM Pro	Solid

Frequency	Analytical Solution	Plate		Solid	
		RF-DYNAM Pro	Ratio	RF-DYNAM Pro	Ratio
f_1 [Hz]	34.344	35.513	1.034	35.481	1.033
f_2 [Hz]	66.046	68.293	1.034	68.238	1.033
f_3 [Hz]	105.673	109.268	1.034	109.666	1.038
f_4 [Hz]	118.882	122.927	1.034	123.172	1.036
f_5 [Hz]	137.375	142.049	1.034	142.131	1.035
f_6 [Hz]	190.212	196.684	1.034	196.672	1.034

Following **Figure 2** shows the first six natural shapes of the investigated plate.

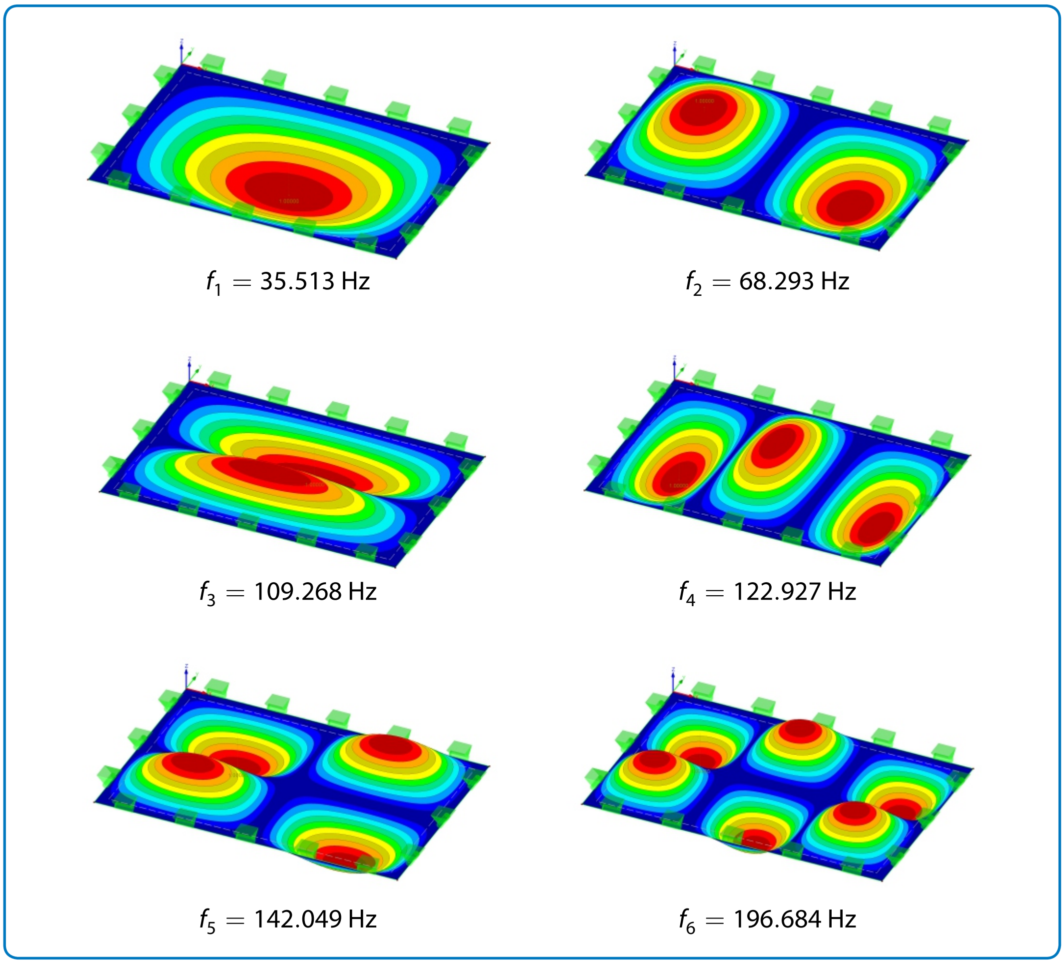

Figure 2: First six natural shapes of the plate in RFEM 5

References

[1] LEISSA, A. W. *Vibration of Plates*. NASA, Washington, D.C..